Jürgen Roth
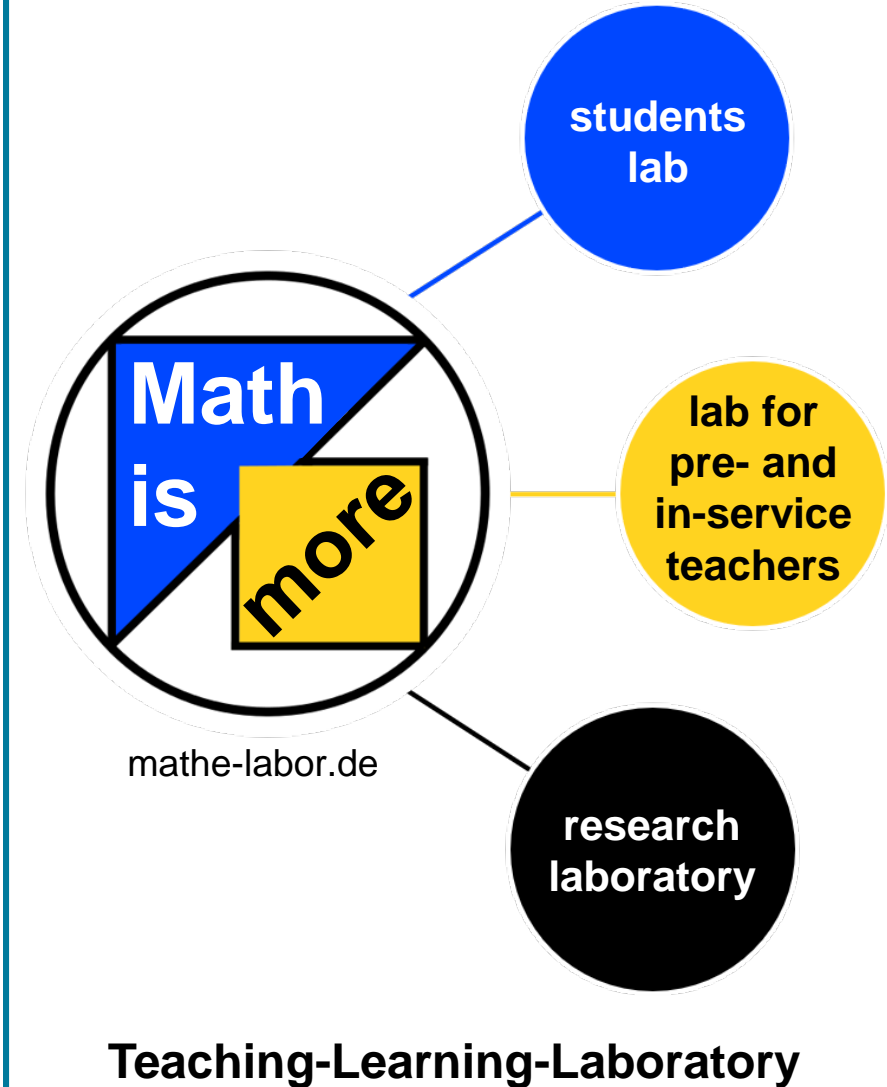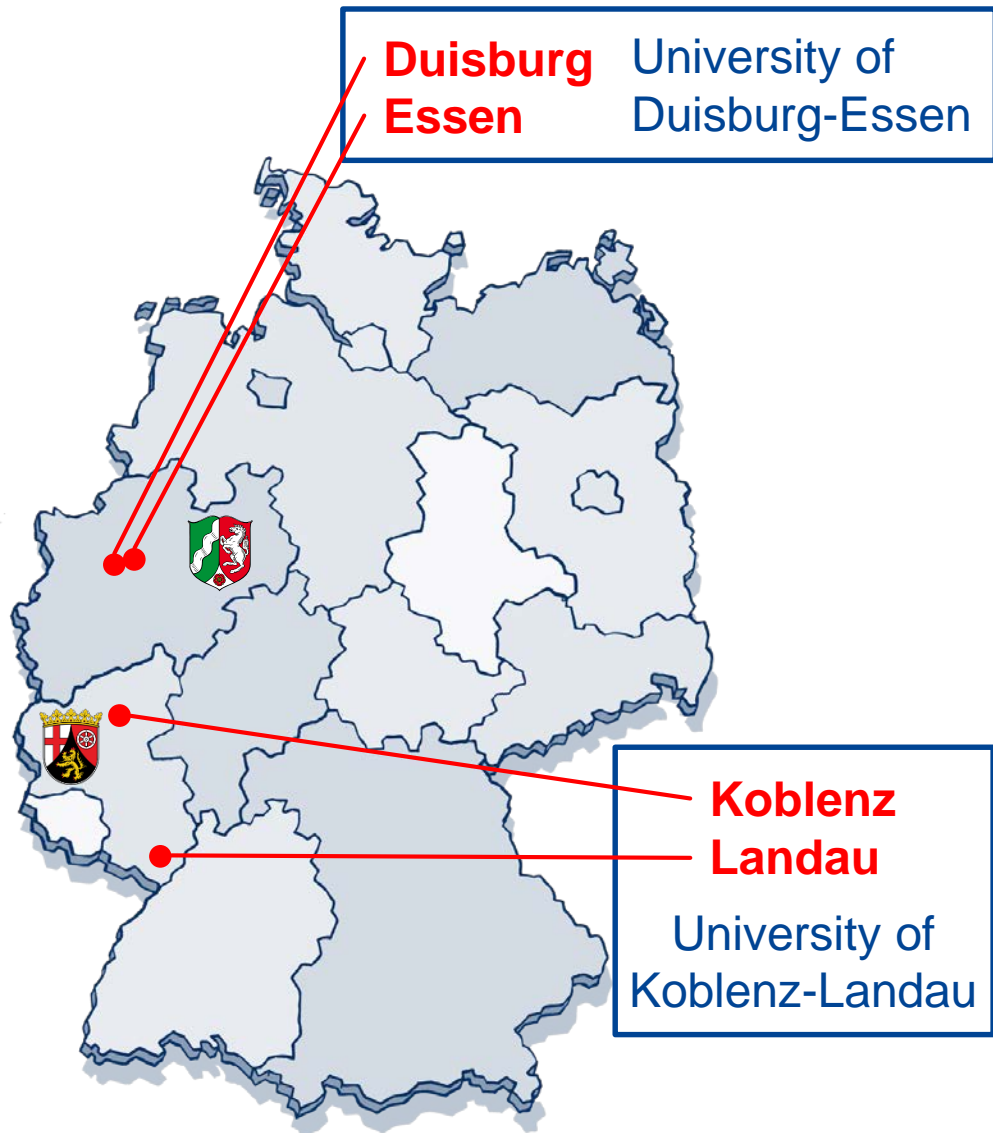
Inspiring teaching and learning of functional thinking by experiments with real and digital materials

Inspiring teaching and learning of functional thinking

1   Inspiring teaching and learning by teaching-learning-laboratories

2   Inspiring learning: Experiments with real or digital materials

3   Inspiring teaching: **Vi**deo **vi**gnettes for the **an**alysis of teaching and learning processes

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Teaching-Learning-Laboratory

Inspiring teaching and learning of functional thinking

1 Inspiring teaching and learning by teaching-learning-laboratories

2 Inspiring learning: Experiments with real or digital materials

3 Inspiring teaching: **Vi**deo **vi**gnettes for the **An**alysis of teaching and learning processes
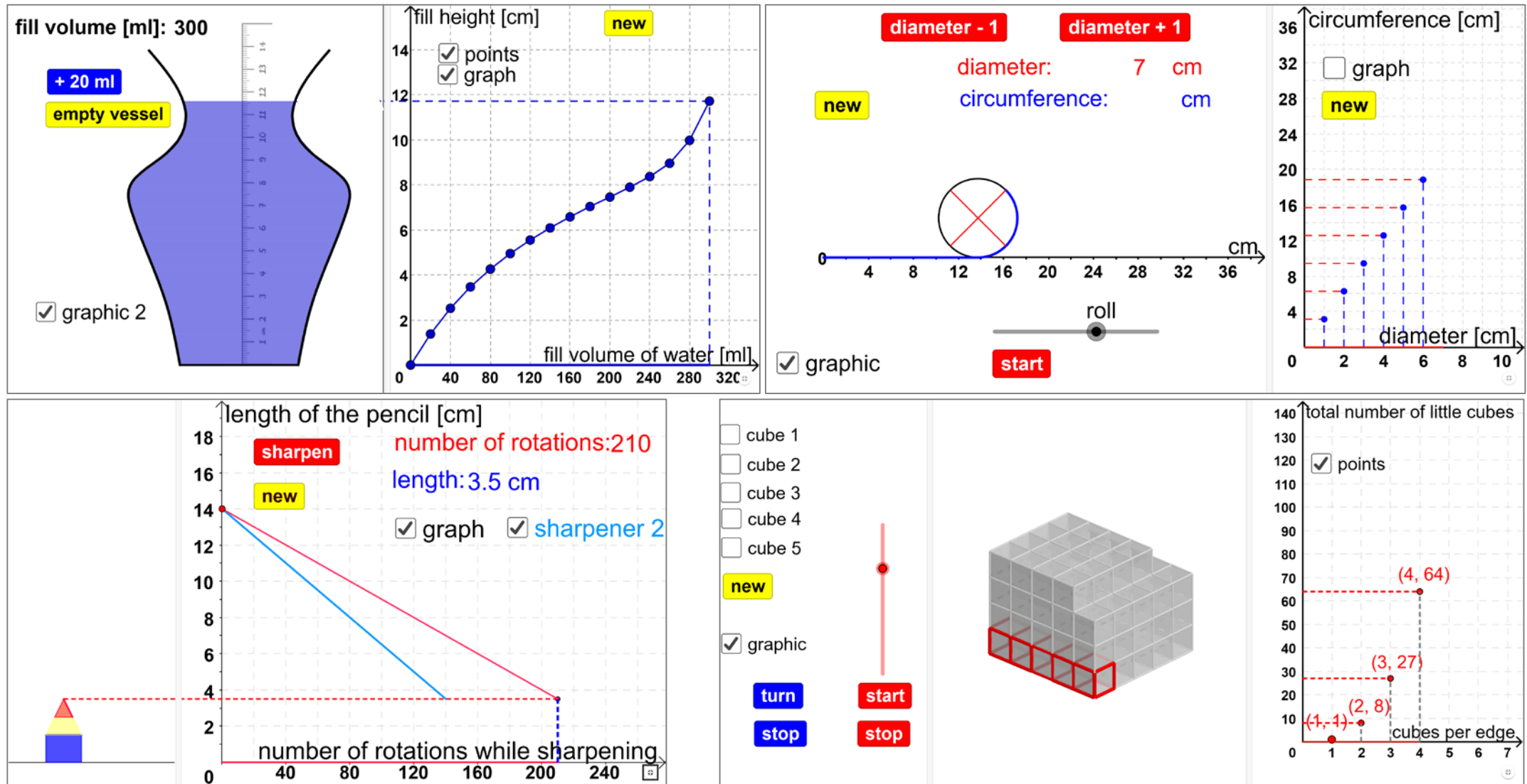
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Selection of situations

- practicability
- comparable actions
- use media advantages
- different functional relationships
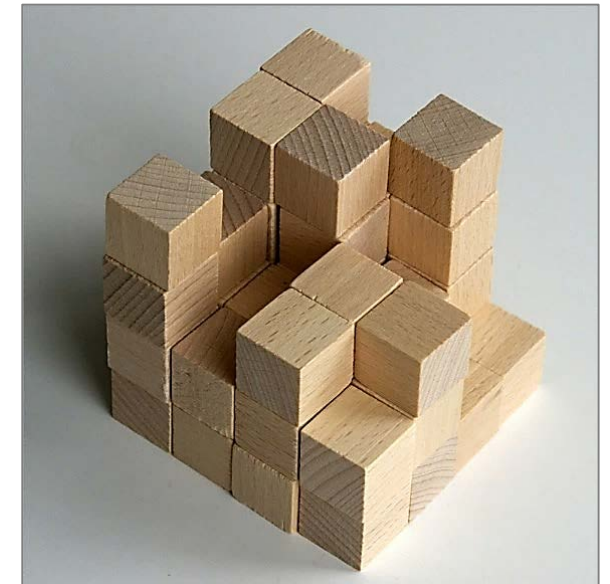
Lichti & Roth 2018

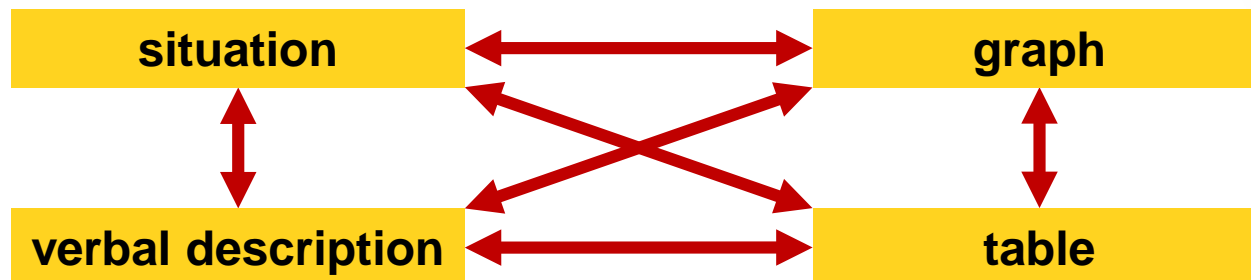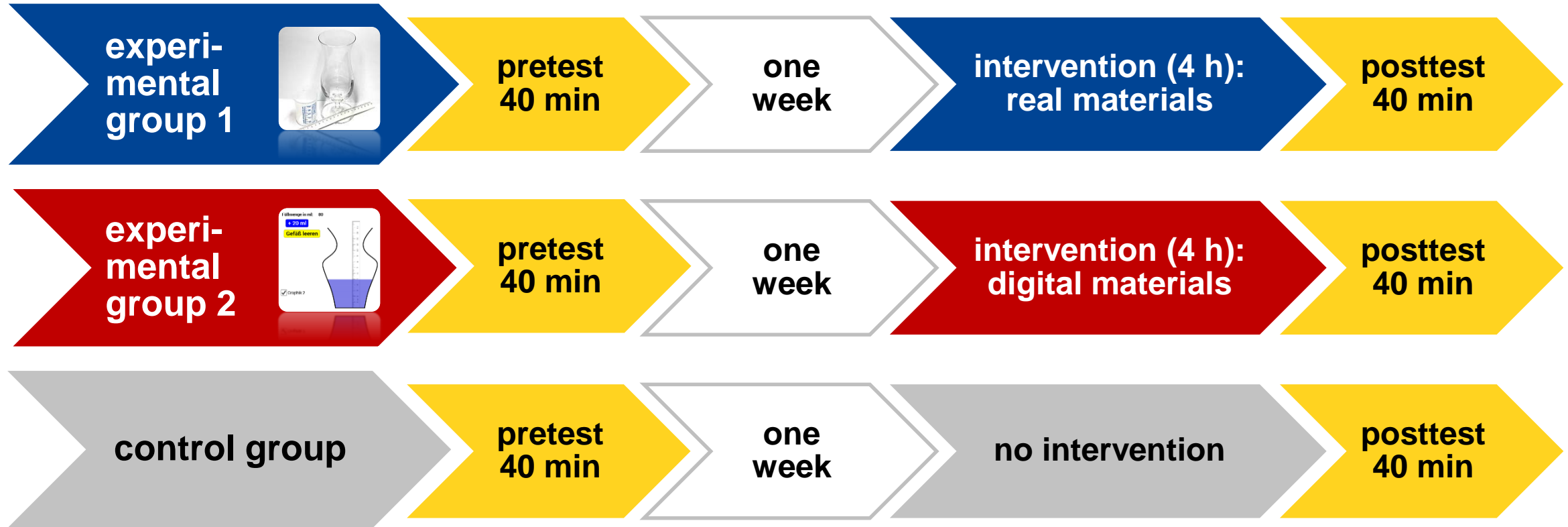www.geogebra.org/m/ncqCZurb

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▶ **Tasks**

▷ Determine values (measurement)

▷ Create tables of values

▷ Working with graphs

    ▶ "real"-group: draw graphs

    ▶ "digital"-group: observe genesis of graphs

    ▶ both groups: additional drawing of and working with graphs

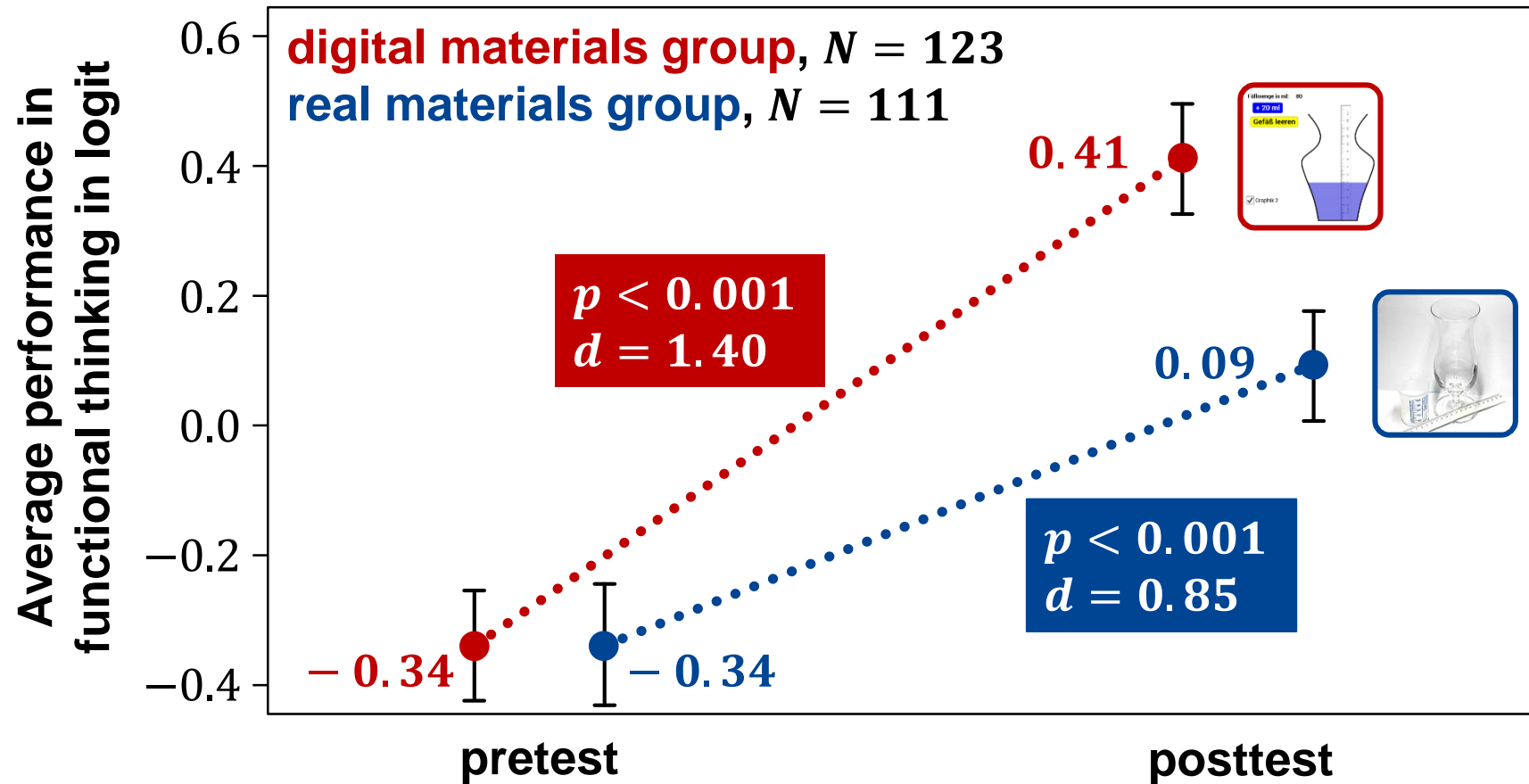▷ Interpolation, application and transfer

▶ **Forms of representation**

situation		graph
verbal description		table

Lichti & Roth (2018, 2019)

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experi-mental group 1		pretest 40 min	one week	intervention (4 h): real materials	posttest 40 min
experi-mental group 2		pretest 40 min	one week	intervention (4 h): digital materials	posttest 40 min
control group	pretest 40 min	one week	no intervention	posttest 40 min

▷ thirteen classes of 6th-graders ($N = 282$)

▷ School year 2015/16 just bevor summer holidays

Lichti, M. & Roth, J. (2019). Functional thinking − A three-dimensional construct? *Journal für Mathematik-Didaktik, 39*, DOI 10.1007/s13138-019-00141-3, https://rdcu.be/bqRk8

digital materials group, $N = 123$
real materials group, $N = 111$

$p < 0.001$
$d = 1.40$

0.41

$p < 0.001$
$d = 0.85$

0.09

-0.34

-0.34

pretest

posttest

Average performance in functional thinking in logit

0.6 0.4 0.2 0.0 -0.2 -0.4

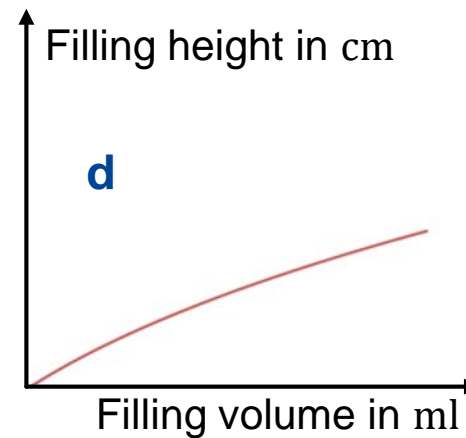$F(1, 25.820) = 8.856, p = 0.006, \eta_p^2 = 0.090$

Reason?

Qualitative content analysis
Tasks from intervention
and posttest with
open response formats

Lichti & Roth (2018, 2019)

Here you can see various vessels and filling graphs.
Match them and justify your choice.

1 **2**

3 **4**

Filling height in cm

a

Filling volume in ml

Filling height in cm

b

Filling volume in ml

Filling height in cm

c

Filling volume in ml

Filling height in cm

d

Filling volume in ml
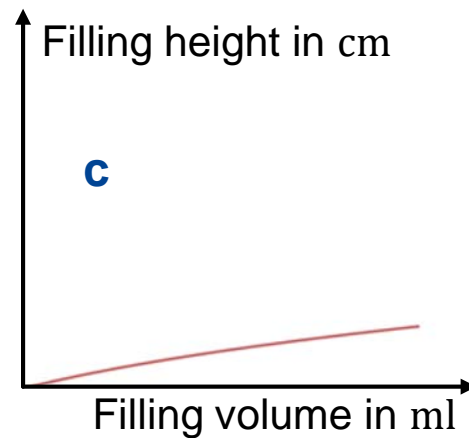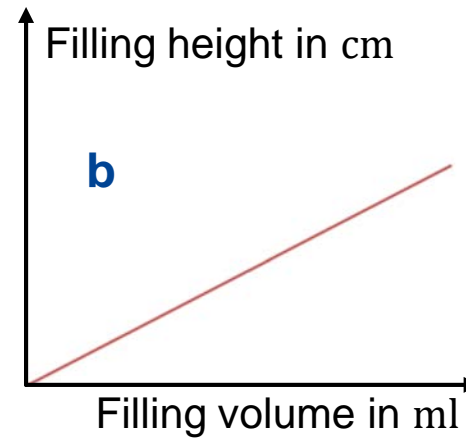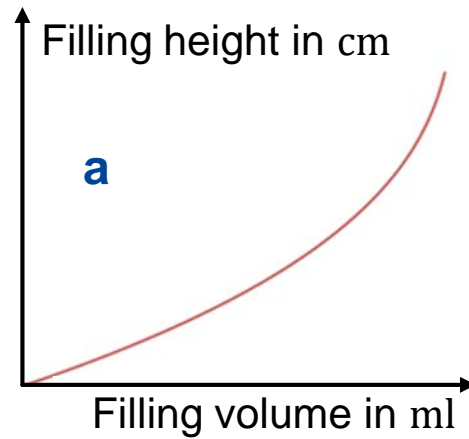
Category	Example
shape of the vessel	The glass is getting wider.
course of the graph	Because the graph is pretty flat.
rise of the water	First the water rises slower, then faster.
state	The graph is steep. The bowl is flat.
variation	The graph becomes steeper and steeper.

Lichti & Roth (2018)

▶ **Real materials group**
Students argue significantly more frequently

  ▷ with the shape of the vessel
    $(\chi^2 = 14.79, df = 1, p < 0.001, V = 0.15)$

  ▷ with states
    $(\chi^2 = 4.361, df = 1, p = 0.037^*, V = 0.08)$

▶ **Digital materials group**
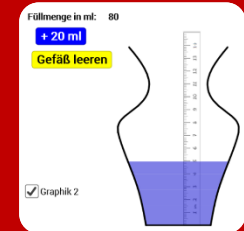Students argue significantly more frequently

  ▷ with the course of the graph
    $(\chi^2 = 6.62, df = 1, p = 0.01^*, V = 0.10)$

  ▷ with variations
    $(\chi^2 = 6.955, df = 1, p = 0.008^{**}, V = 0.11)$

Real materials:
Real situation
and state

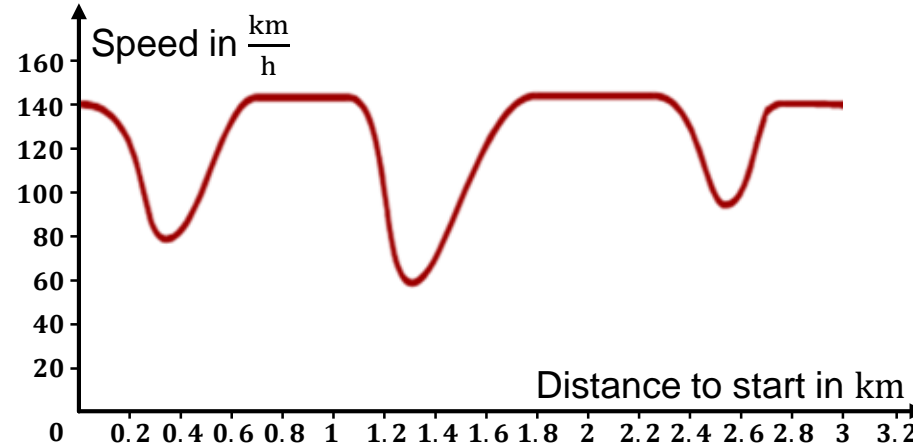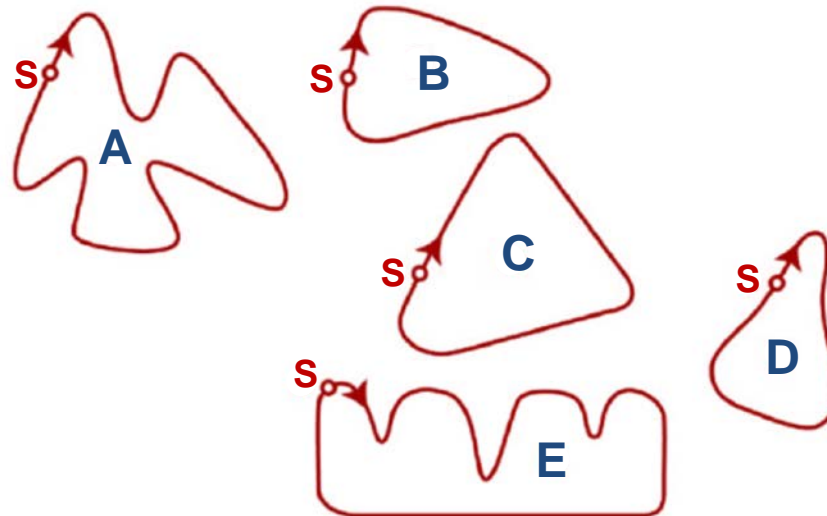**Digital materials:**
Graphical
representation
und variation

Lichti & Roth (2018)

This graph shows how a racing cars' speed changes during the second lap of a racing track that is 3 km long and flat.

Speed in $\frac{km}{h}$

Distance to start in km

Which racing track did the car drive on? The pictures are bird's eye views of the tracks.

☐ **A**
☐ **B**
☐ **C**
☐ **D**
☐ **E**

Justify your choice.

S: Starting line

Category	Example
speed and curves	A car must slow down in curves.
different curves	There are three curves: two flat and one steep.
graph-as-picture error	Because the track looks like the graph.
Incorrect interpretation of the racing tracks	The racetrack [A] has three curves.

Lichti & Roth (2018)

▶ **Essential for the solution**

  ▷ Graph ⇒ three different curves

▶ **Real materials group**

  ▷ Students are significantly more frequently **not** able to apply their knowledge of the relationship between speed and curves.
  $(\chi^2 = 6.304,\ df = 1,\ p = 0.012^*,\ V = 0.21)$
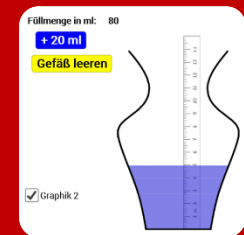
  ▷ graph-as-picture error: 36%

▶ **Digital materials group**

  ▷ graph-as-picture error: 0%

Real materials:
graph-as-picture
error

Digital materials:
link graph and
real situation

Lichti & Roth (2018)

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▶ **Quantitative analysis**   **materials**

  ▷ Development of functional thinking   **digital** **>** real

▶ **Qualitative analysis**   **materials**

  ▷ Argumentation with graphs   **digital** **>** real

  ▷ Argumentation with real situations   digital **<** **real**

  ▷ Argumentation with covariation   **digital** **>** real

  ▷ Argumentation with mapping   digital **<** **real**

  ▷ qualitative approach to a graph
    (interpreting and/or drawing)   **digital** **>** real

  ▷ Single pairs of values
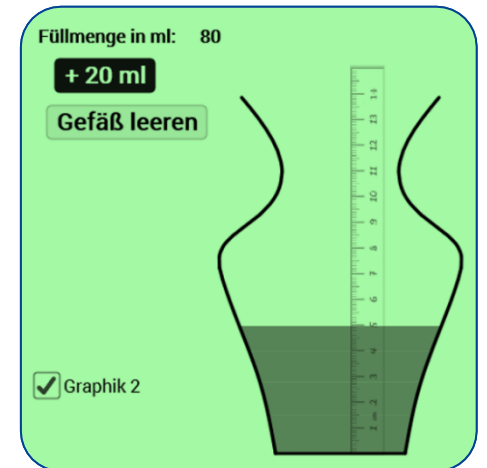   (interpreting and/or drawing)   digital **<** **real**

Lichti & Roth (2018)

Inspiring teaching and learning of functional thinking

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      teaching and learning processes

Bartel & Roth (2017)

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Bartel & Roth (2017a)

Student level

Learning Environment: Topic and Goals

Meta level

ViviAn ✕

Functions

Terms

Fractions

Area and volume

Logout

Vignette 1

Vignette 2

Vignette 3

Vignette 4

Vignette 5

Vignette 6

S3

Student profile

S2 S3
S1 S4

Temporal classification

Diagnostic tasks

00:51 ▬▬▬ 02:52

Bartel & Roth (2017), Enenkiel & Roth (2017), Hofmann & Roth (2017), Walz & Roth (2017)

http://vivian.uni-landau.de

Task

Materials

Student documents

Learning Environment: Topic and Goals

Meta level

Student profile

	S2	S3	
S1			S4

Temporal classification

Diagnostic tasks

00:51 ▬▬▬▬ 02:52

Diagnostic tasks: Video vignette 1

2. Describe the problems that occur in the videotaped situation.

Student level

Task

Materials

Student documents

Diagnostic tasks:
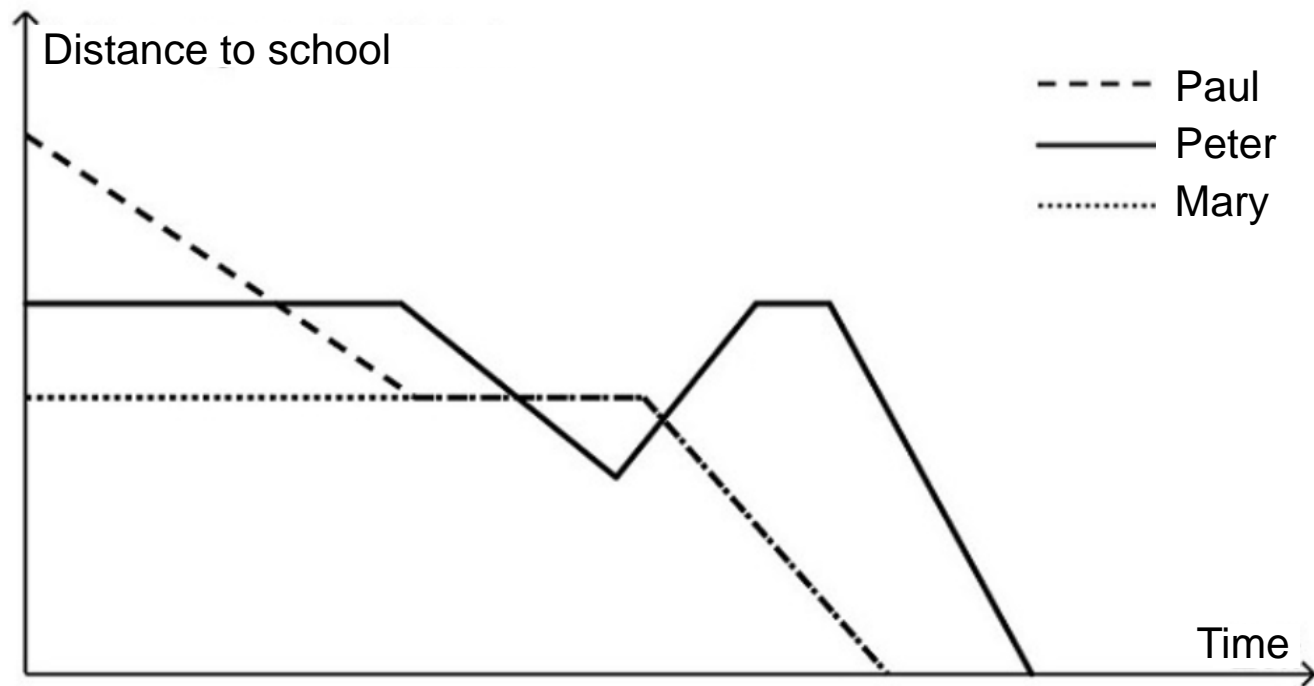
2. Describe the pro

ViviAn

Task

Peter, Paul and Mary go to school and live in the same street. At the end of the street is their school. Every morning they walk to school. The picture shows where the three were yesterday at different times.

Write a story about Peter's way to school yesterday.

close

Materials

Distance to school

- - - - Paul
——— Peter
·········· Mary

Time

close

Student level

Task

Materials

Student documents

Learning Environment:

Materials

Distance to school

- - - - Paul

——— Peter

·········· Mary

Time
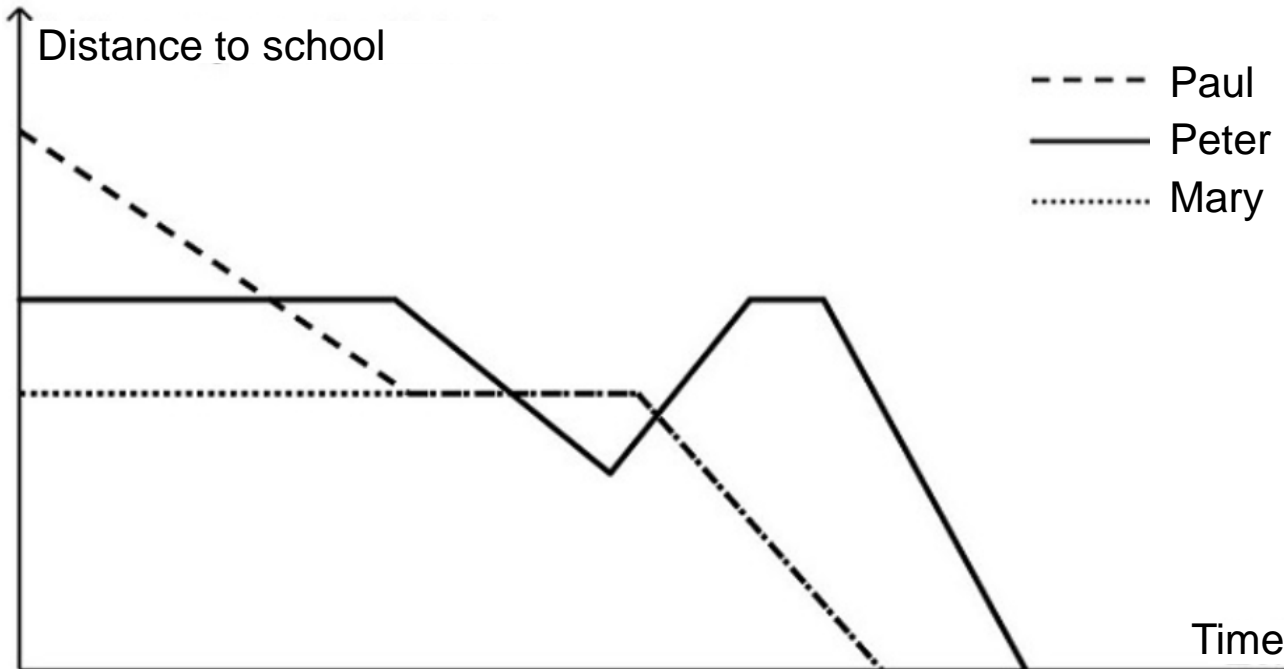
close

Student documents

Student 1 | Student 2 | Student 3 | Student 4

Eines Tages ging Peter x ganz alleine zur Schule. Er schoss ein Stein vor sich her. Peter geht zur Schule und muss an eine Ampel warten, dann geht er weiter muss aber einen Umweg wegen eine Baustelle machen dan steht er kurz wieder an einer Ampel. Zum Schluss beeilt er sich da er nicht zu spät kommen will.

Student 1 | Student 2 | Student 3 | Student 4

Peter steht an der Ampel, wartet dort, dann lief er los. Jetzt nimmt er einen Umweg wegen einer Baustelle. Jetzt ist er wieder kurz an einer Ampel und lief schnell zur Schule

close

Learning Environment: Topic and Goals

Task

Materials

Student documents

Student profile

S2 S3

S1   S4

Temporal classification

Diagnostic tasks

ViviAn

00:51 ▬▬▬▬ 02:52

Diagnostic tasks: Video vignette 1

2. Describe the problems that occur in the videotaped situation.

In some cases, the students fail to grasp the relations represented in the graph (e.g. axis labeling).

Learning Environment: Topic and Goals

Task

Materials

Student documents

Student profile

	S2	S3	
S1			S4

Temporal classification

Diagnostic tasks

S3

▶ 00:51 ▬▬▬▬▬ 02:52 🔊 ⛶

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Diagnostic tasks: Video vignette 1

2. Describe the problems that occur in the videotaped situation.

You replied:

In some cases, the students fail to grasp the relations represented in the graph (e.g. axis labeling).

Experts have given the following answers:

- At first, not all pupils correctly recognize the context depicted. At times, the "speed" is considered instead of the

Diagnostic tasks

▷ Work through student tasks.

▷ Describe observations.

▷ Interpret observations and give reasons for interpretations (basic ideas, student (mis)concepts, ...)

▷ Suggest and justify teaching activities.

Diagnostic tasks: Video vignette 1

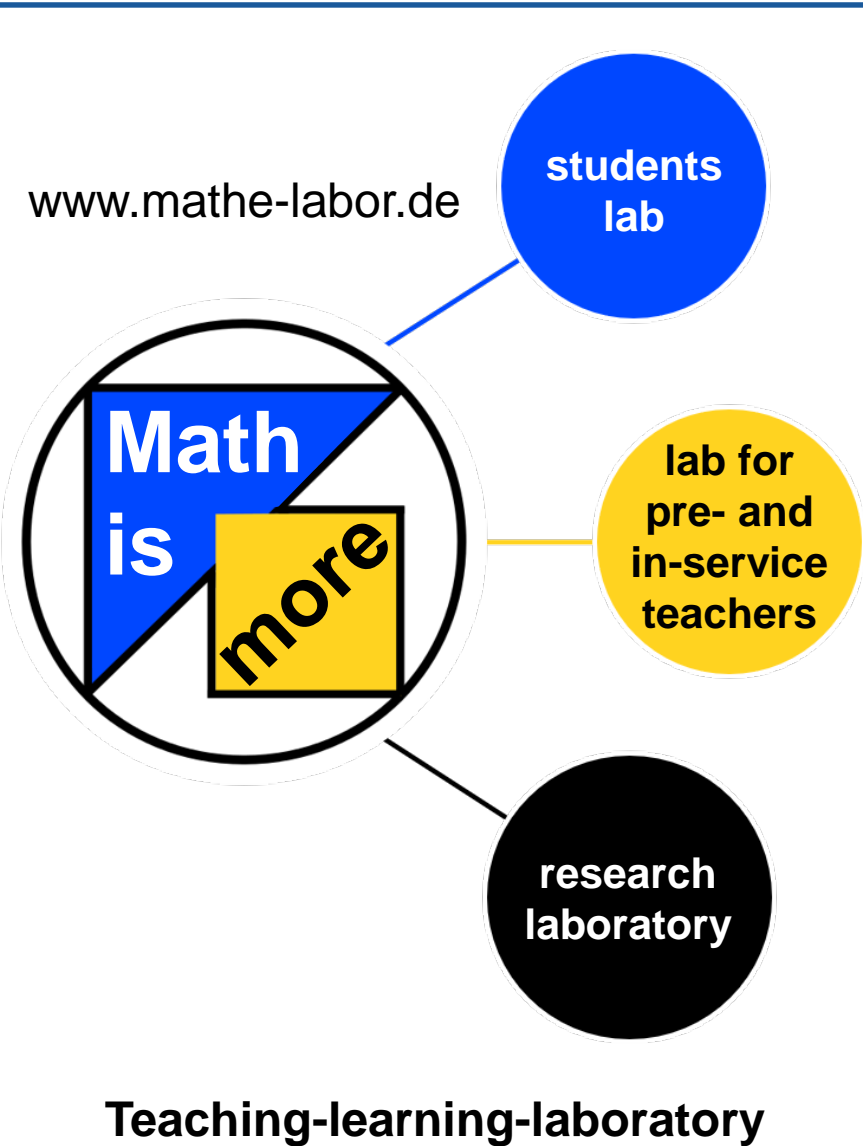2. Describe the problems that occur in the videotaped situation.

You replied:

In some cases, the students fail to grasp the relations represented in the graph (e.g. axis labeling).
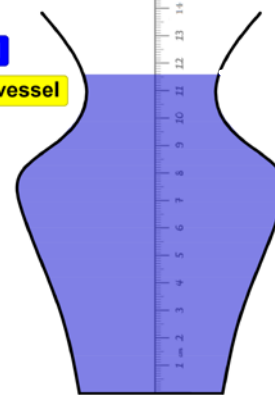
Experts have given the following answers:

- At first, not all pupils correctly recognize the context depicted. At times, the "speed" is considered instead of the "distance from school".
- A negative gradient of the graph is interpreted as a higher speed than a positive gradient (although the sign of the gradient at this point has no influence on the speed).
- The students do not understand what the horizontal sections of the graph mean. They assume that there is still distance covered in this section, but the speed does not change.
- It is not recognized where the school is in the coordinate system.
- The graph-as-picture error occurs because it is argued that as the graph rises, the person goes up a mountain and thus slows down.
- Uncertainty as to which concept of time is used in the task: The time the students still need until they reach school or the "normal" time that goes on while something is happening.

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www.mathe-labor.de

students lab

Math is more

lab for pre- and in-service teachers

research laboratory

Teaching-learning-laboratory

fill volume [ml]: 300

+ 20 ml

empty vessel

Learning environment

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Student level
- Task
- Materials
- Student documents

Learning Environment: Topic and Goals

S3

Meta level
- Student profile
  - S2 S3
  - S1 S4
- Temporal classification
- Diagnostic tasks

00:51 — 02:52

ViviAn

Video **vi**gnettes for the **An**alysis of teaching and learning processes

ViviAn

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Thank you for your attention.

Mathematics Education
Secondary Level

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