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Kinematical Thinking – an Important Process Goal in Mathematics Education

What is kinematical thinking? – An approach

In advertisements for all kinds of jobs one can find the requirement of "mental mobility" (German: "Bewegliches Denken"). Since – in a first approach – the expression allows various interpretations, the following is an attempt to describe and specify it for didactical purposes in mathematics education. In psychology various facets of "mental mobility" are distinguished. E.g. Aebli (1983) based on Piaget gives a list of aspects whose core statements are as follows[1]:

- The most elementary form of mental mobility (…) is *the ability to reconstruct changes in ones mind.*
- Mobility means (…) *the ability to change the point of view* and to recognize that *the perspective* differs from point to point.
- The planning of actions requires (…) mobility. The actor should *adapt his action plans to the changing conditions*. He should be able to change the intermediate objectives if necessary in order to reach the goal on a new path.

In connection with the expression "mental mobility" Aebli uses several times the word *perspective* and associates with it a discrete change of the point of view. In addition to that another aspect seems to be important for an approach to the term "mental mobility" concerning mathematics education. The question is in which way the changes of the viewpoints take place.

In order to distinguish the didactical term "Bewegliches Denken" in our context from the psychological meaning we use the expression "kinematical thinking". To understand this term it is helpful to interpret "thinking" in the sense of the Greek word for "to think", i.e. *noein* (*noein*). *noein* originally means "to see" und in addition to that "to realize the true facts behind a cover". In a first approach "kinematical thinking" thus manifests itself in the ability to realize an apparently static phenomenon kinematical, or in other words to "see" it mobile in ones mind. This gives the chance to realize additional aspects of the phenomenon.

Thought-processes are obviously beyond control of researchers and cannot be evaluated directly. The oral or written results of a thought-process however often allow conclusions on the thought-patterns or thought-structures which led to them.

[1] Translation by the author.

In the following, components of kinematical thinking are indicated. Depending on the problem, the borders between these components are fluid.

Components of Kinematical Thinking
• "see mobile" and argue • grasp and analyse • realize the kind of variation

"See mobile" and argue: This means, as already mentioned, the ability to realize an apparently static phenomenon kinematical or in other words to "see" it mobile in ones mind. This gives the chance to realize additional aspects of a phenomenon. Moreover this component of kinematical thinking includes the ability to use this imagined movement for argumentations while solving problems. The ability to only mentally visualize the movement of a single object e.g. a vertex of a triangle in a plane is not yet kinematical thinking.

Grasp and analyse: A Part of kinematical thinking is also the competency to grasp a real or imagined movement and analyse its implications for the whole configuration. This includes the ability to change the focus on certain aspects of a figure (e.g. various angles) and thus to concentrate on relevant invariants respectively variables.

Realize the kind of variation: The third substantial aspect of kinematical thinking is the ability to realize and describe the way in which the variation takes place. It includes the competency to describe quantitatively the way of variation in problem solving situations. This can be used e.g. to make predictions about the solutions of equations.

Where does kinematical thinking play a role?

Mobile thinking, in the sense of the components mentioned above, can be used in mathematics e.g. to understand or interpret terms, to increase the awareness of problems, to solve problems and to prove.

In the didactical literature (in German language) one can find references to singular components of kinematical thinking at the latest since the "Meraner Reform" (see Krüger 2000). In current didactical publications one can still find aspects of kinematical thinking quite often e.g. in relation with functional thinking, space perception, movement, kinetics/dynamics, variation and aspects of variables. At each case abilities are needed which correspond with one or more of the components of kinematical thinking mentioned above. Although the necessity of kinematical thinking for the understanding of concepts is obviously (at leased implicitly) recognized and demanded locally in many fields, a global strategy for the development and promotion of kinematical thinking in instruction is missing.

My work bases on the following

Thesis: Kinematical thinking provides another (additional) access to definitions, concepts, problem solving and proofs.

The outcome of this thesis for my work is the following

Objective: Conception and evaluation of learning environments that develop and use kinematical thinking profitably in mathematics education.

If the thesis above is accepted, it has to be a substantial process goal of mathematics instruction to develop pupils' kinematical thinking. An important application of kinematical thinking is the following: If learners fail to use or lack a calculus for solving a problem, kinematical thinking gives the students the possibility for an alternative approach to it.

Learning Environments

The possibility to visualize mathematical concepts with computer programs offers a new chance to develop kinematical thinking. It has to be ensured however that learners do not just "push the mouse" but deal actively with the invariants and variables that change. Under the suitable measures to reach this goal are in particular the demand of oral and written verbalisations of forecasts, explanatory statements and findings.

It is important to point out two substantial objectives: After the work with the computer programs learners have to be able

- to grasp, analyse and realize the way of variation of similar movements *without computers* or other resources and
- in case of more complex situations to plan and structure a suitable use of computer programs.

Figure 1 represents an Excel-sheet that has been embedded into a learning environment and designed as an experimental material for the area of *terms*. It has been conceived to develop a conception of the way of variation of terms.

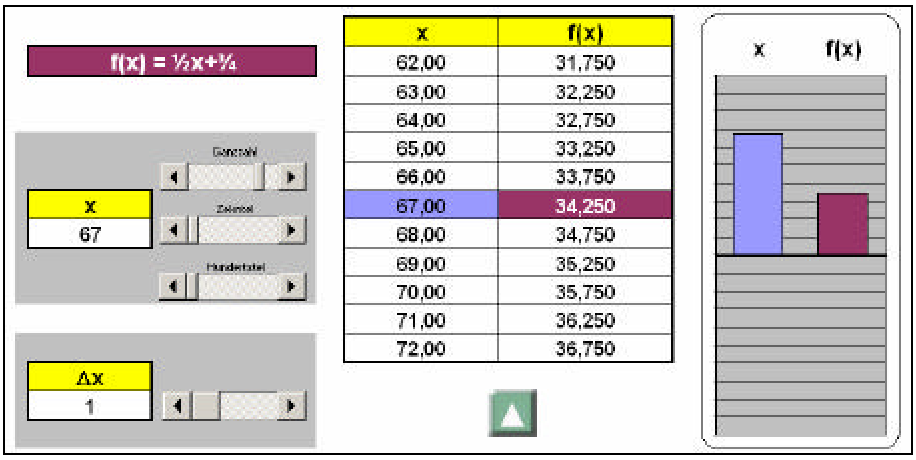

Figure 1

Figure 2 represents the following problem: A circle k and two points A and C on it are given. The length of the chord $e = [AC]$ obviously depends on the positions of the points A and B. To be able to deal with those positions a coordinate plane is introduced with the circle's centre as point of origin. The positions of the points A and C are given as their x-coordinates relating to this coordinate plane. At first the point A is kept fix and the length of e is observed, while the point C is being dragged around the circle. In the left diagram in figure 1 the length of the chord e is represented as a function of the x-coordinate of the point C $[e(x_C)]$.

- Consider why the path of the point LeC looks like represented.
- Try to find out, what happens with the path of LeC, if you drag the point A along the circle.

As already mentioned the length of chord e depends on the positions of both points A and C. Therefore it seems to be natural, to visualize these dependencies simultaneously. The right diagram in figure 2 shows the surface which results of the equation $z = e(x_C; x_A)$.

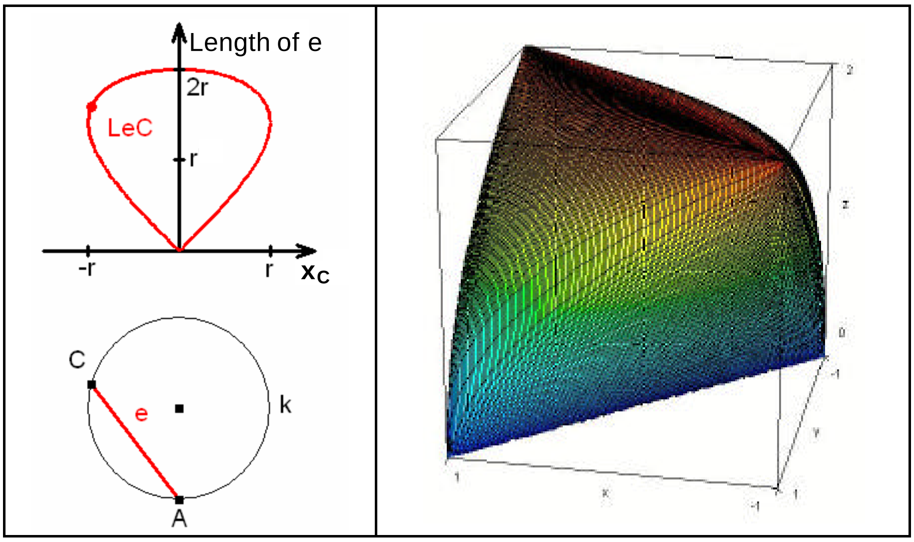

Figure 2

Literature

Aebli, Hans: *Zwölf Grundformen des Lehrens.* Klett-Cotta, Stuttgart (1983), 2001[11]

Krüger, Katja: *Erziehung zum funktionalen Denken – Zur Begriffsgeschichte eines didaktischen Prinzips.* Dissertation vorgelegt am Fachbereich Mathematik der J. W. Goethe-Universität Frankfurt im Juli 1999, Logos-Verlag, Berlin, 2000