Gleichdicks

Ein interessantes Phänomen mit vielfältigen Bezügen

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Didaktik der Mathematik Sekundarstufen

Gleichdicks

1. Das Phänomen Gleichdicks
2. Räumliche Gleichdicks
3. Anwendungen

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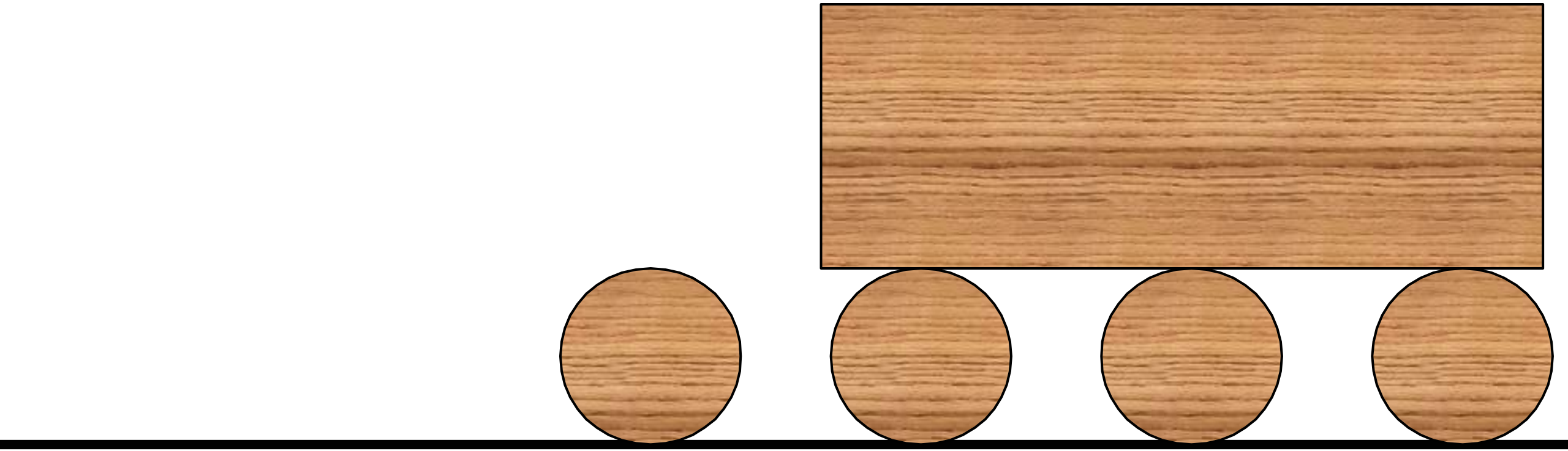
1 Das Phänomen Gleichdicks

Unterlegrollen



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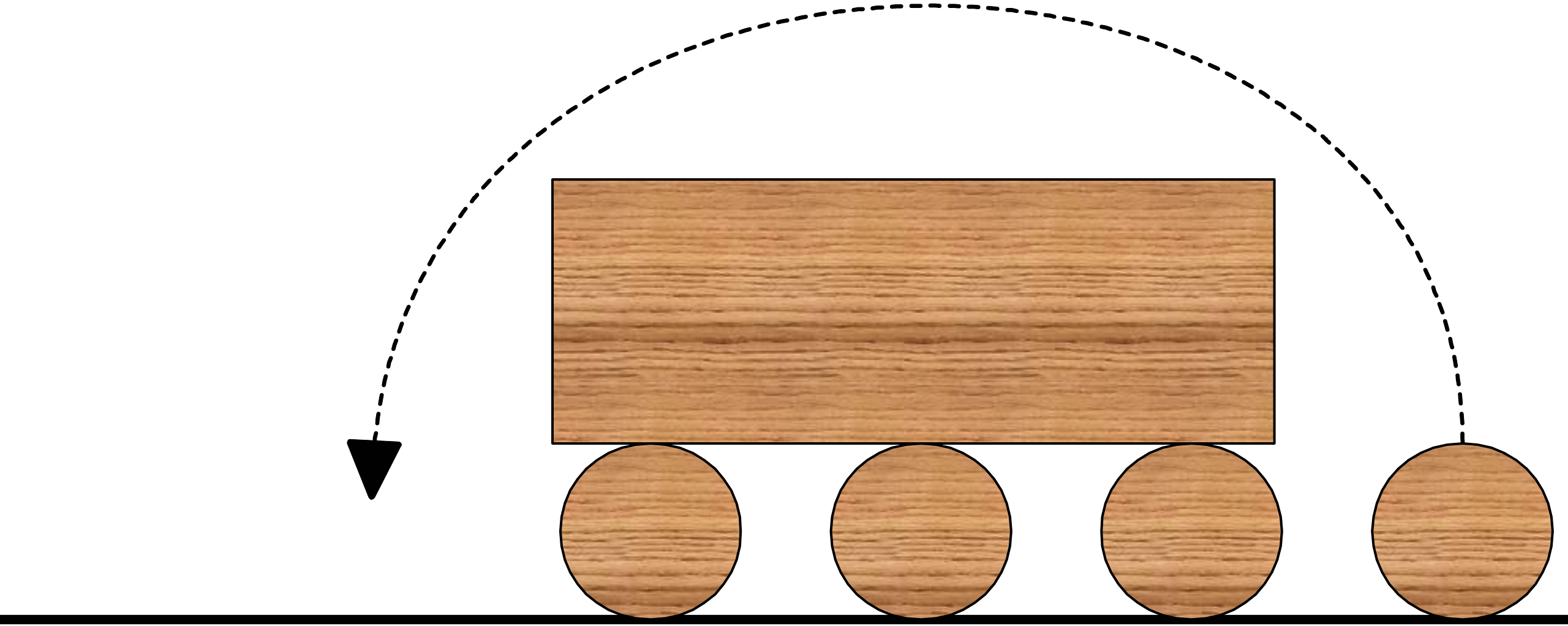

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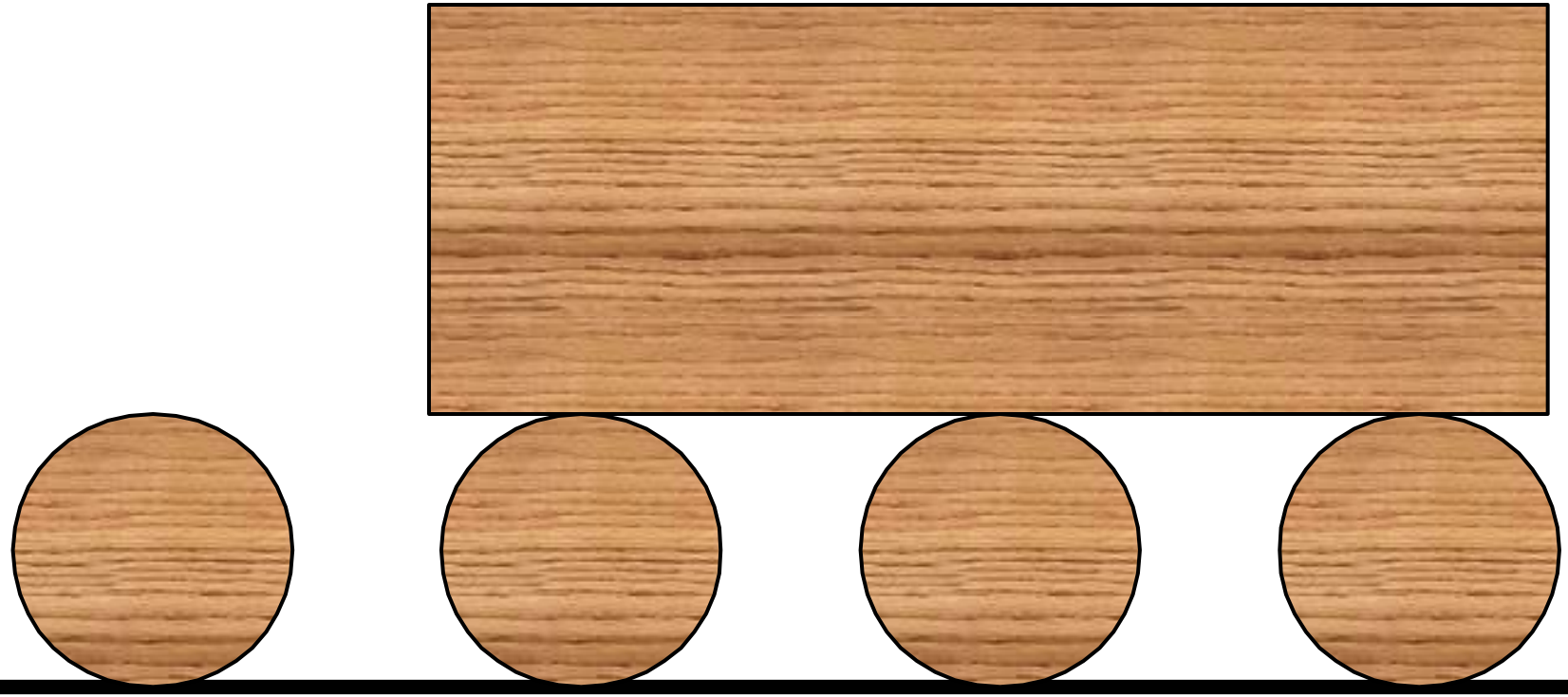

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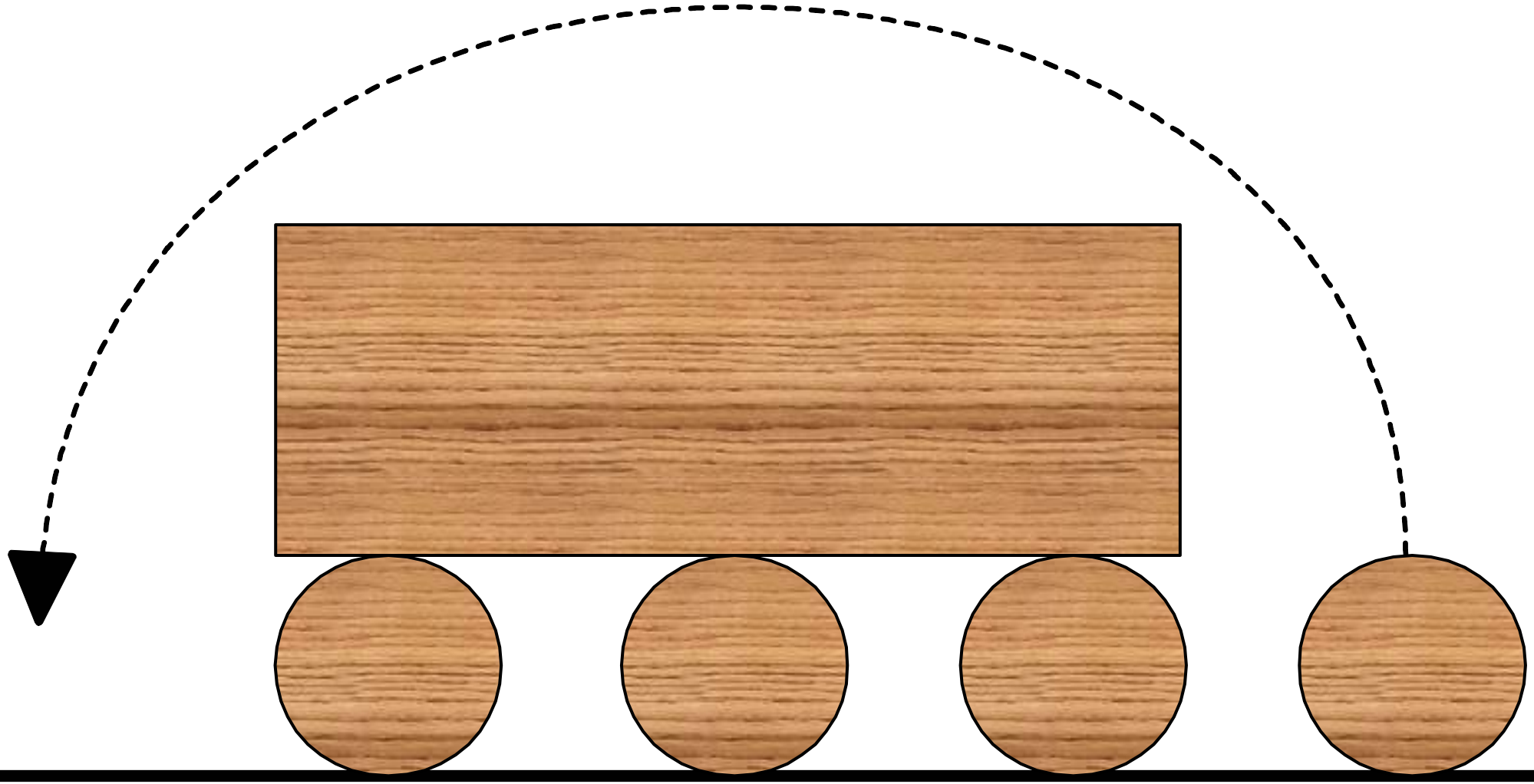

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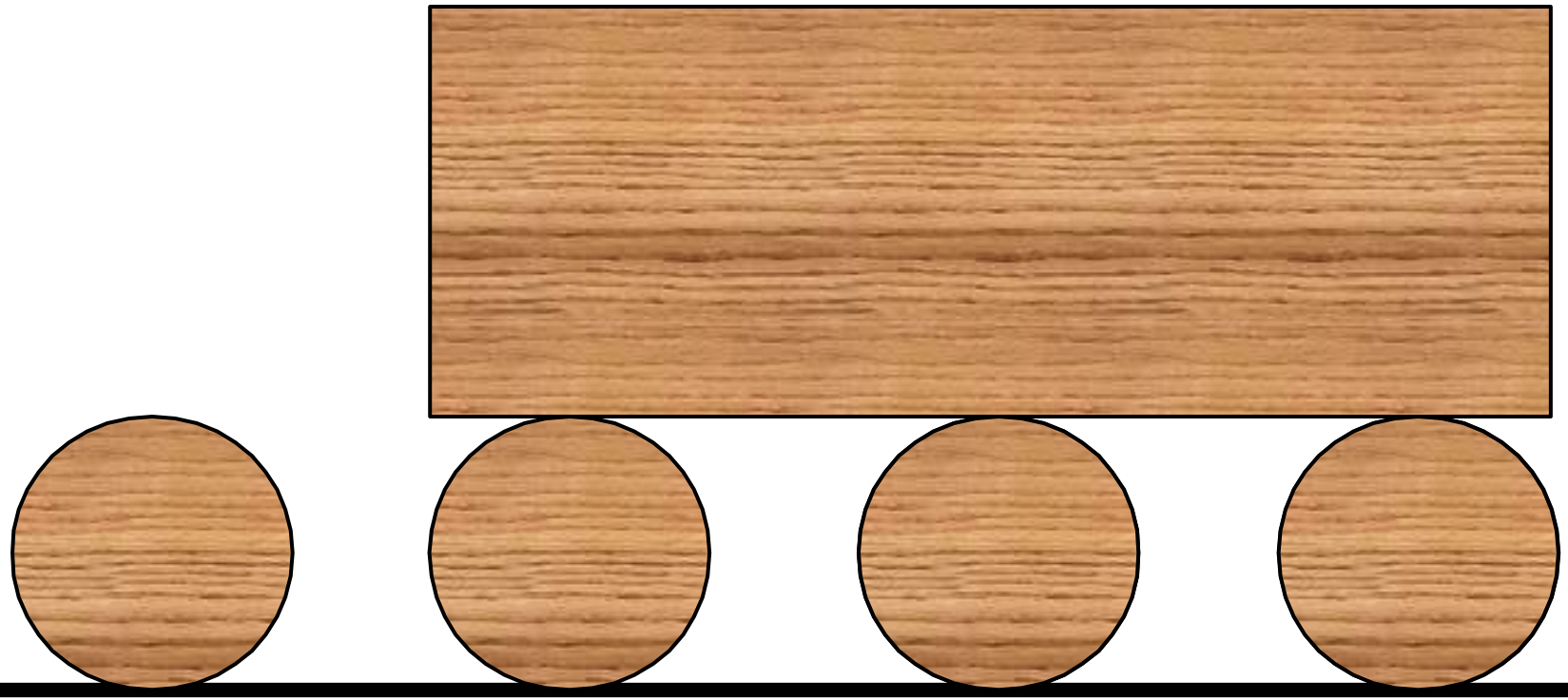

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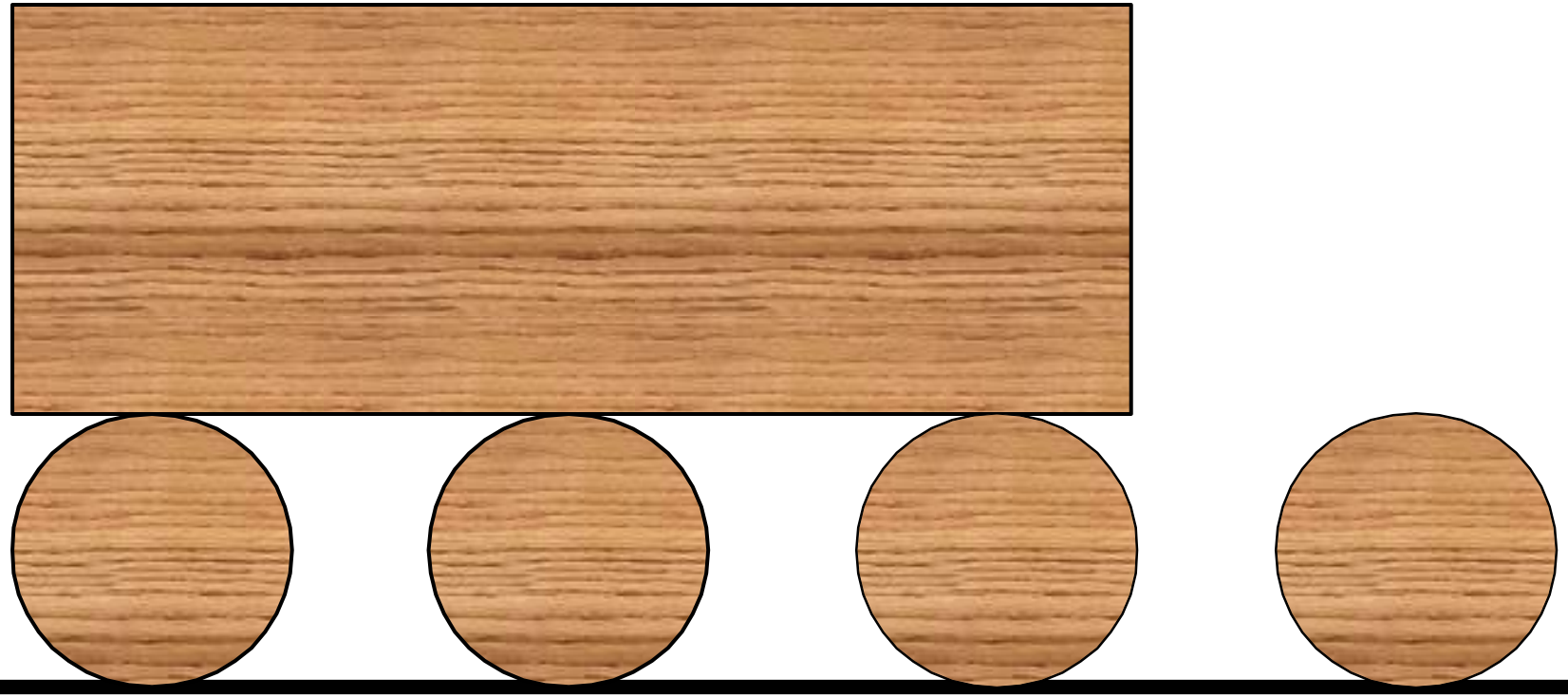

Unterlegrollen?

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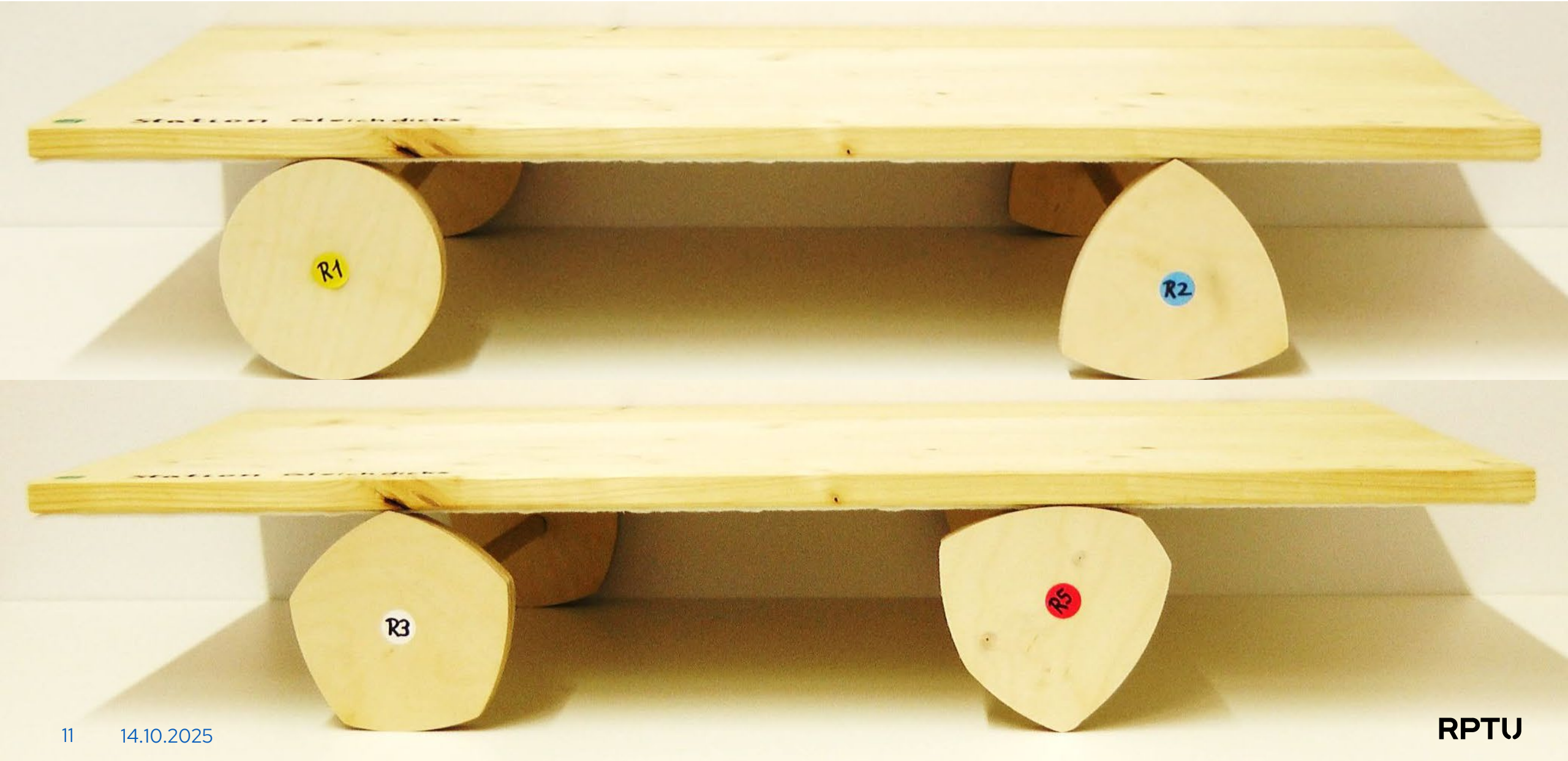
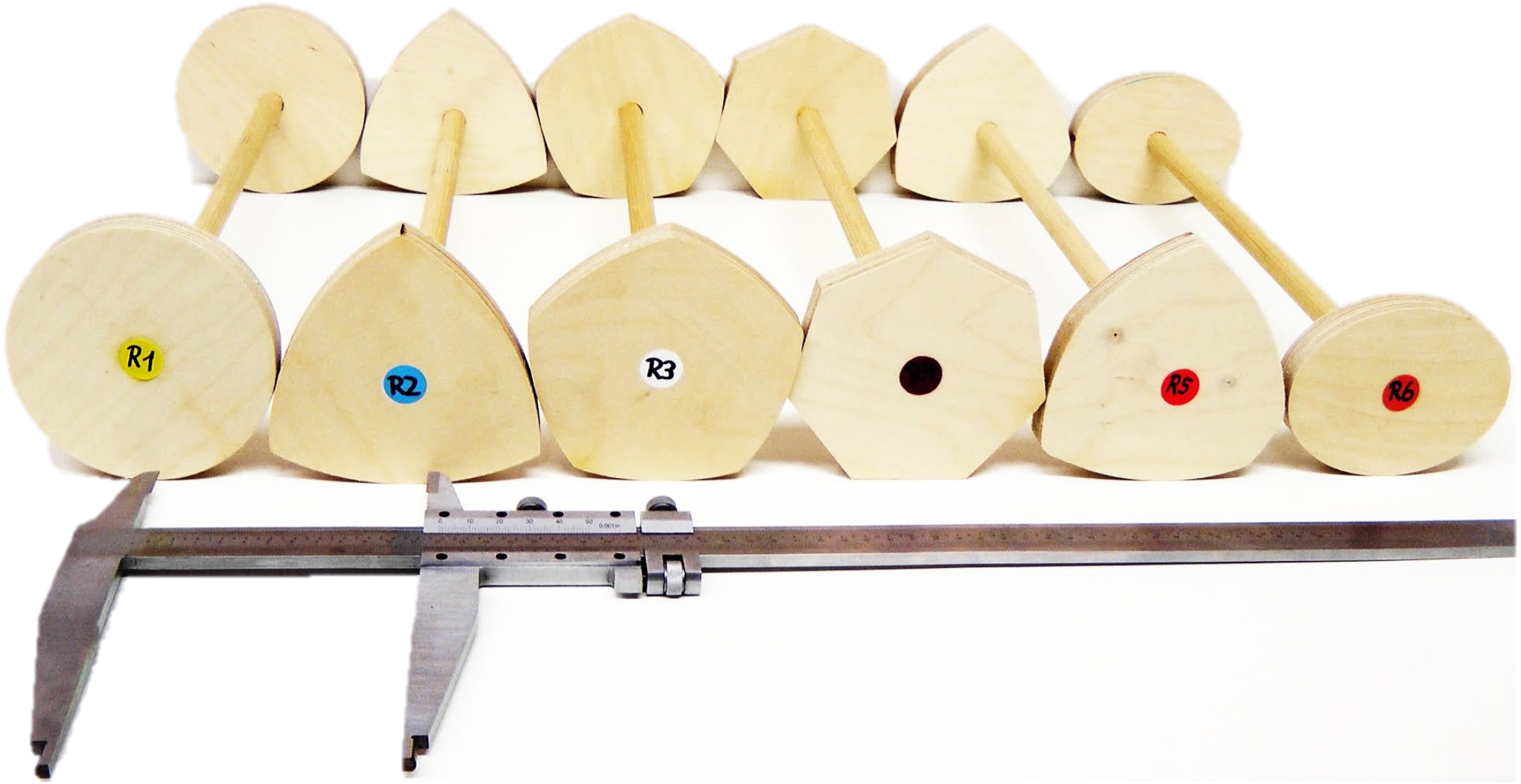


Gleichdicks?



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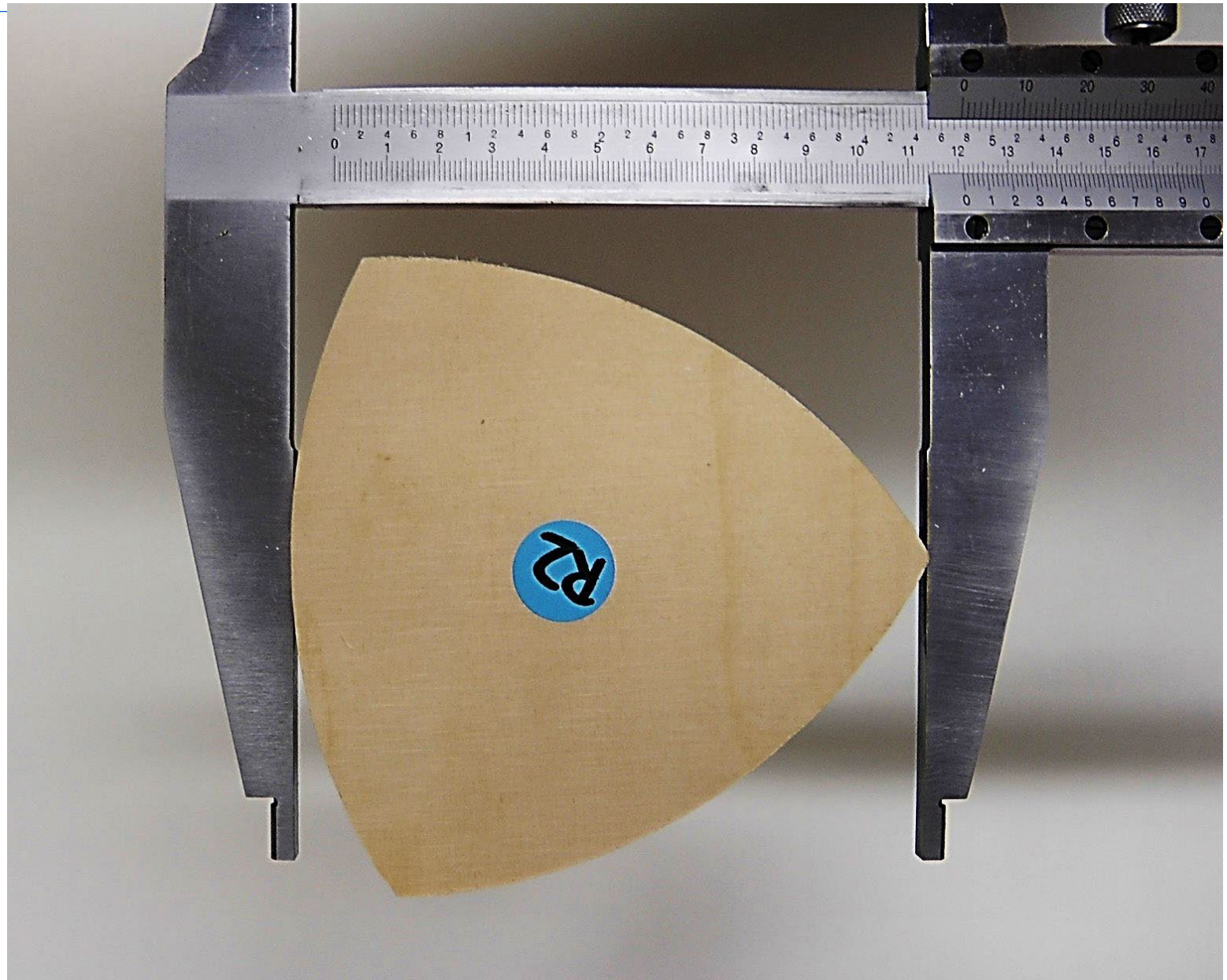
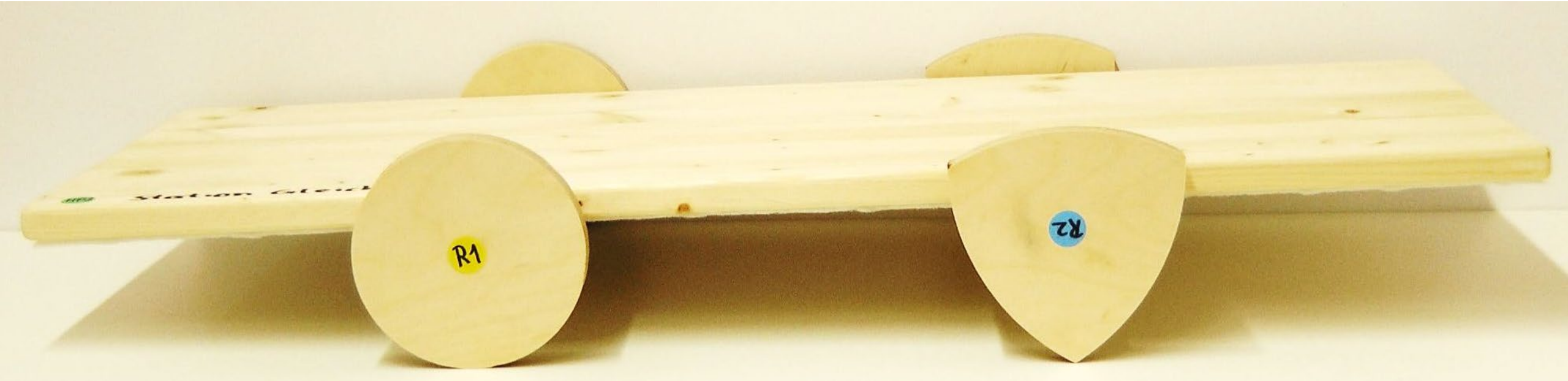
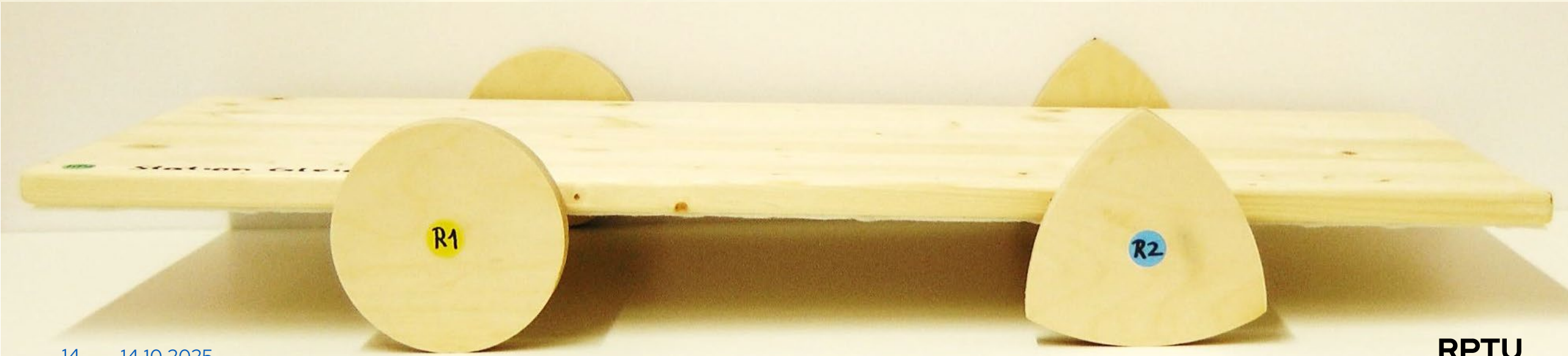
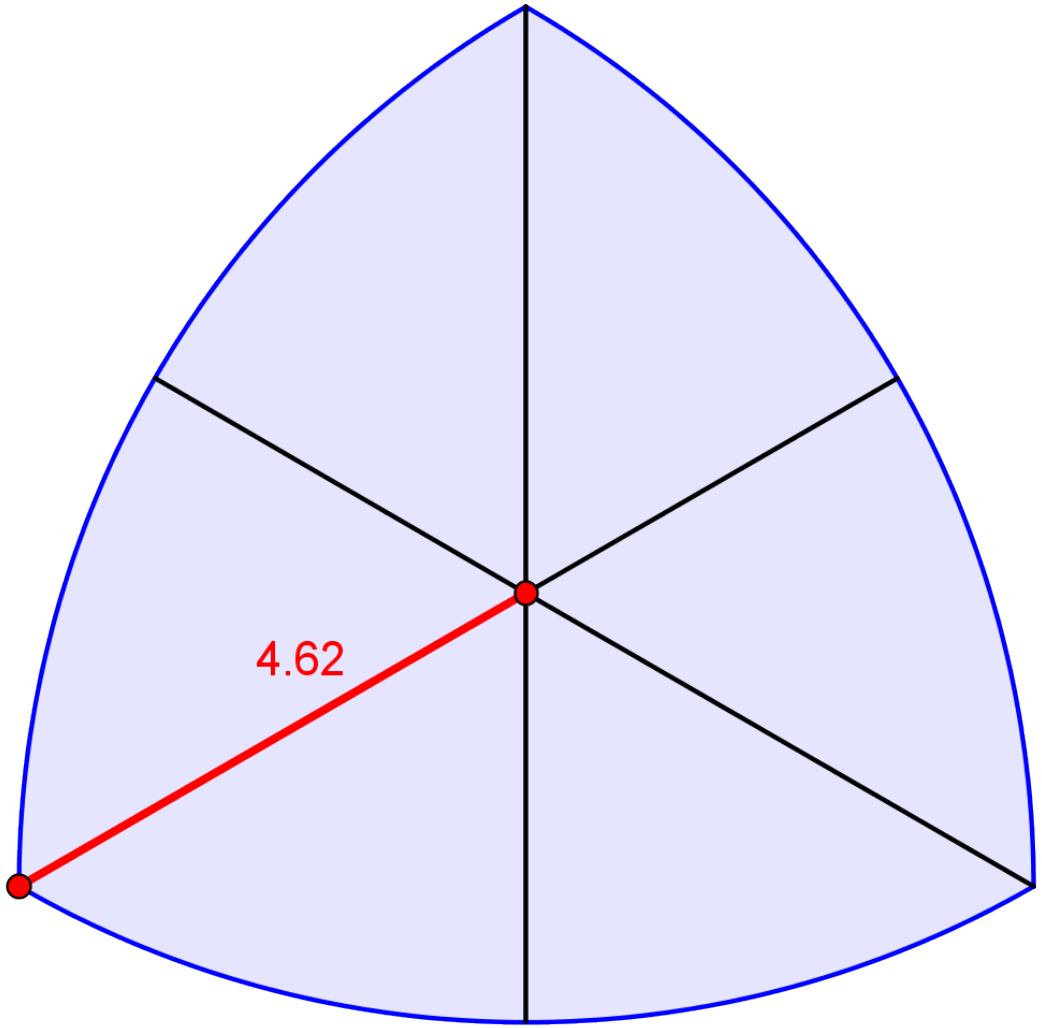


Räder an Achsen?





Grundvorstellung: Speichen



Dicke:

d = 8

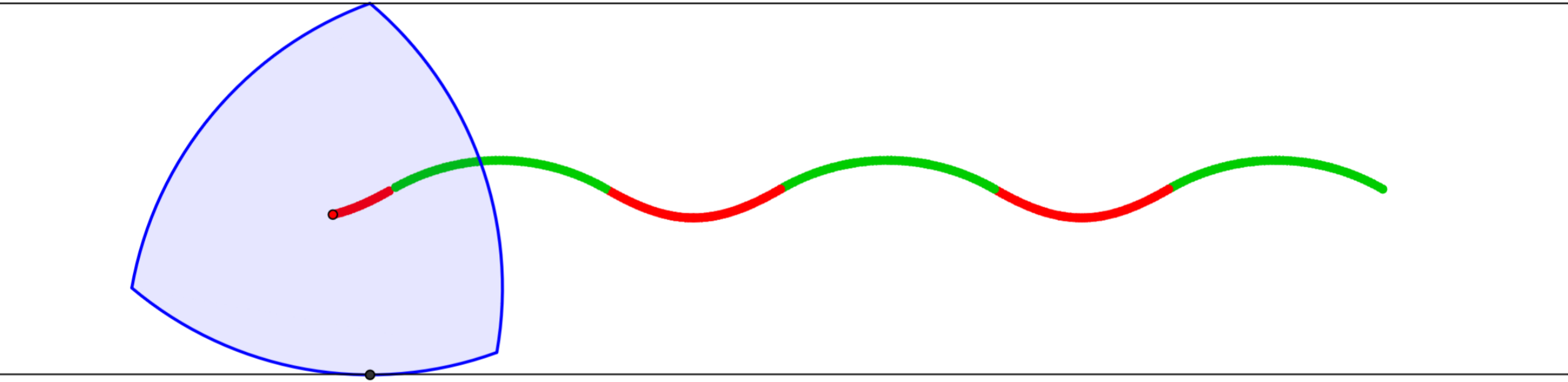
Figur auswählen:

- Reuleaux-Fünfeck (RF)
- Reuleaux-Dreieck (RD)
- Kreis

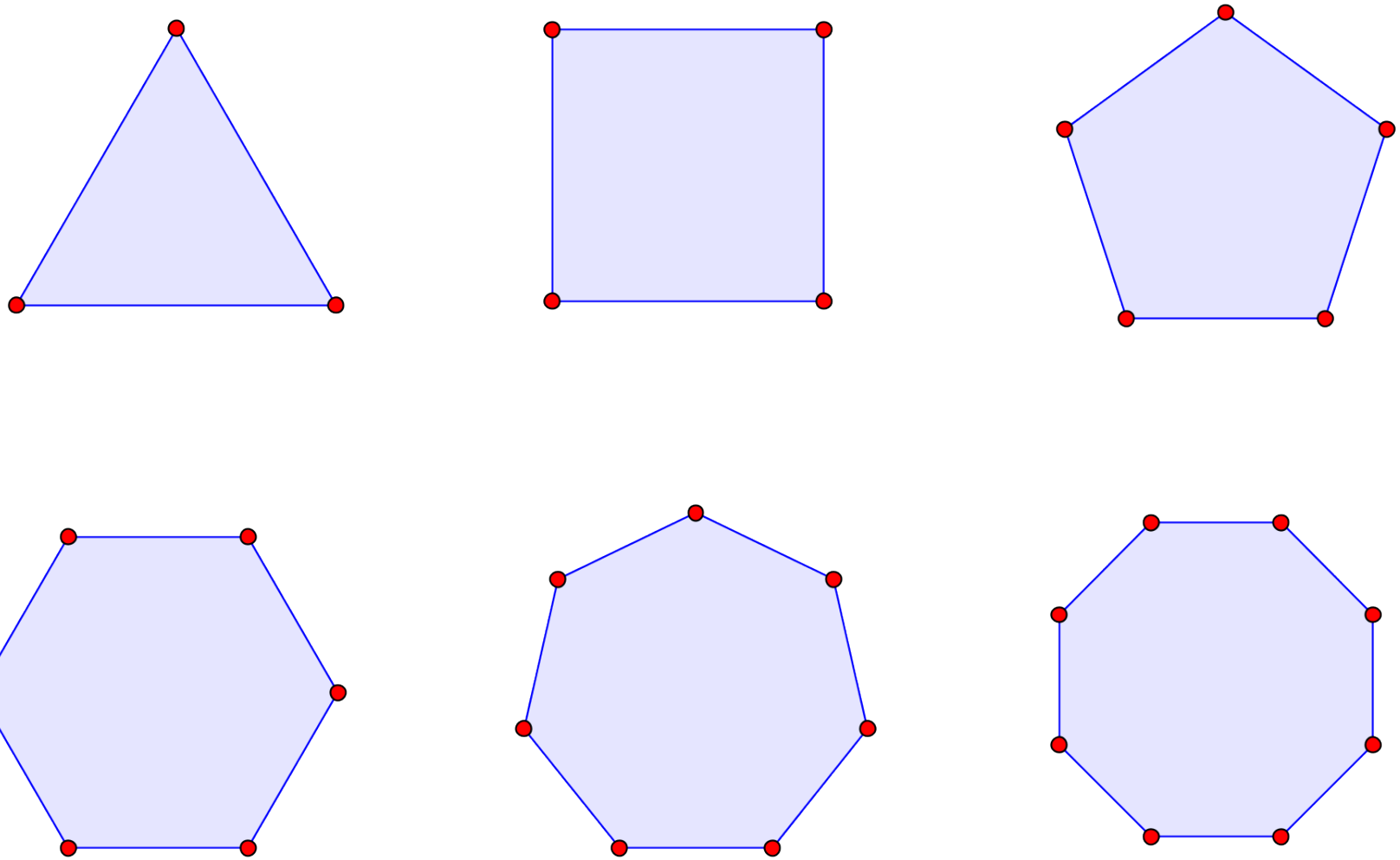
Speichen messen:

Speiche rotieren

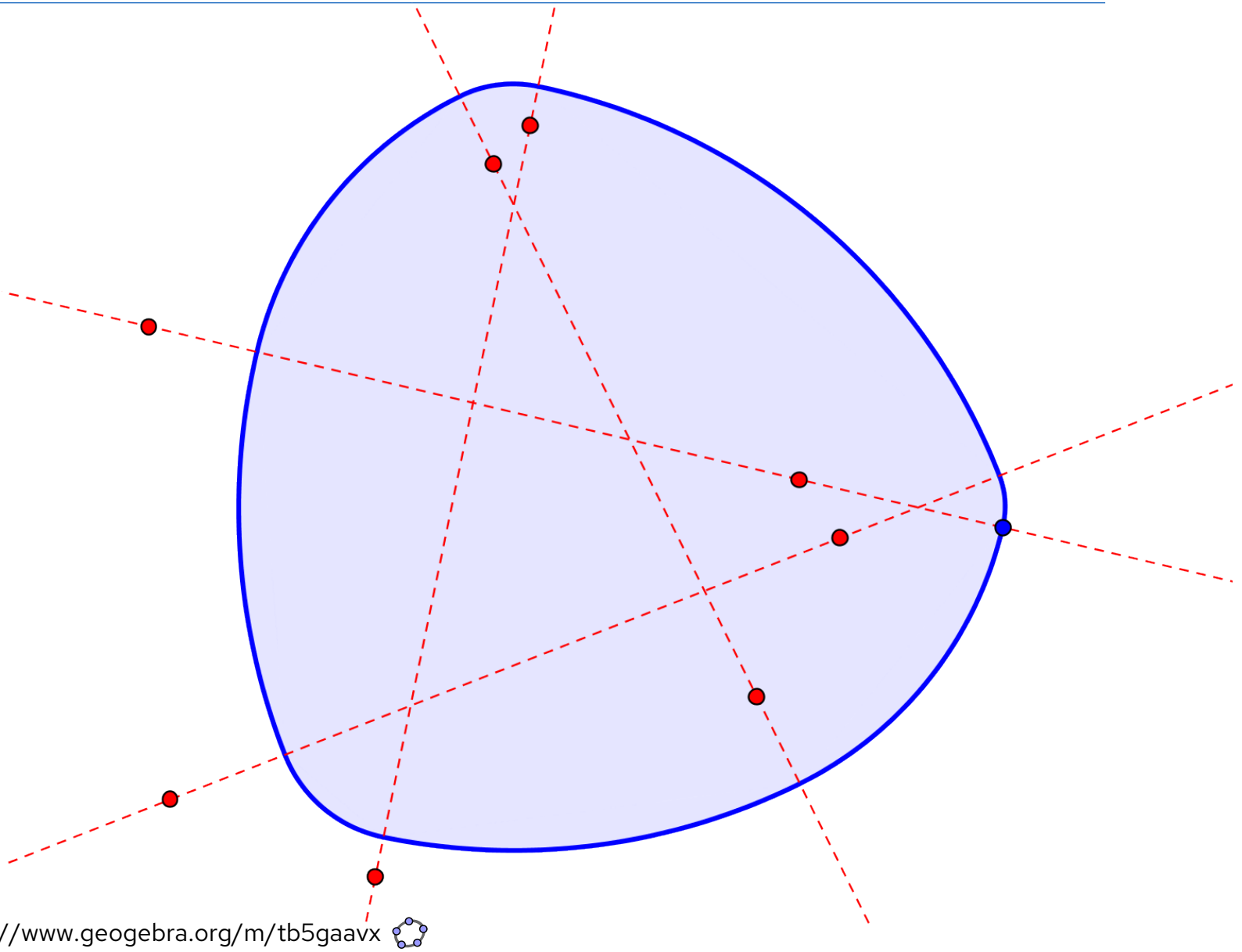
Radachsen beim Gleichdick



Regelmäßige Gleichdicks konstruieren

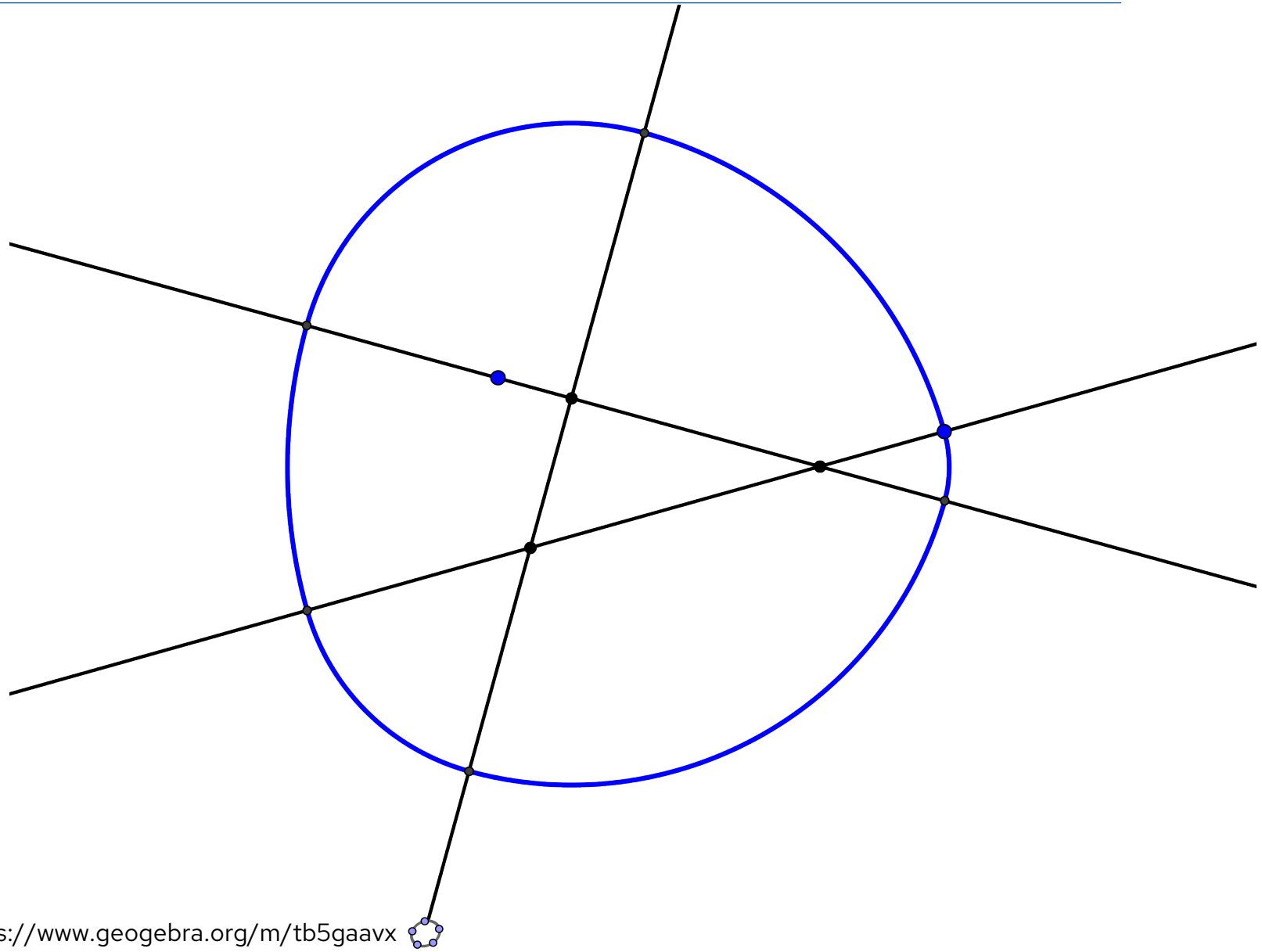


Unregelmäßige Gleichdicks



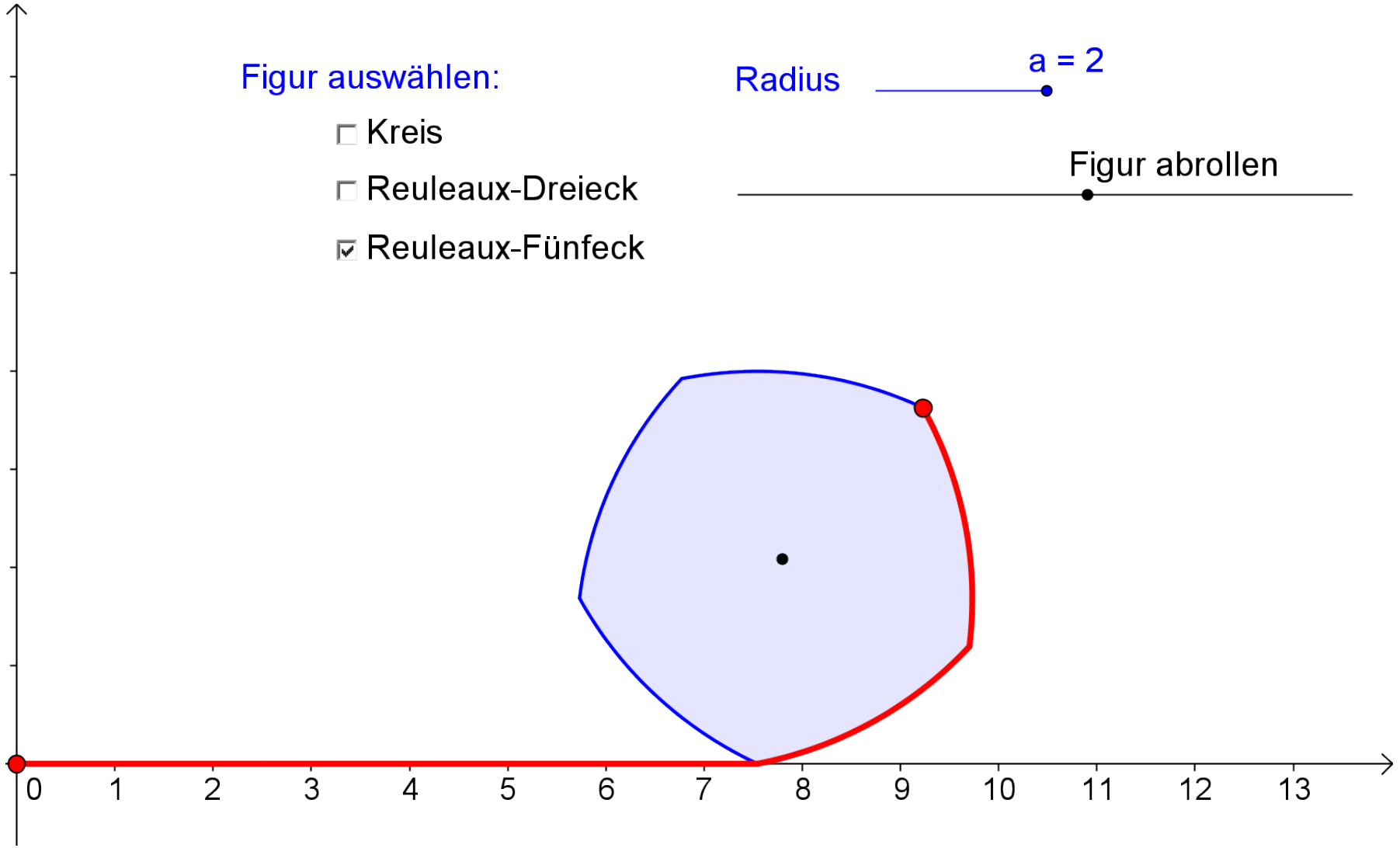
https://www.geogebra.org/m/tb5gaavx

Unregelmäßige Gleichdicks konstruieren

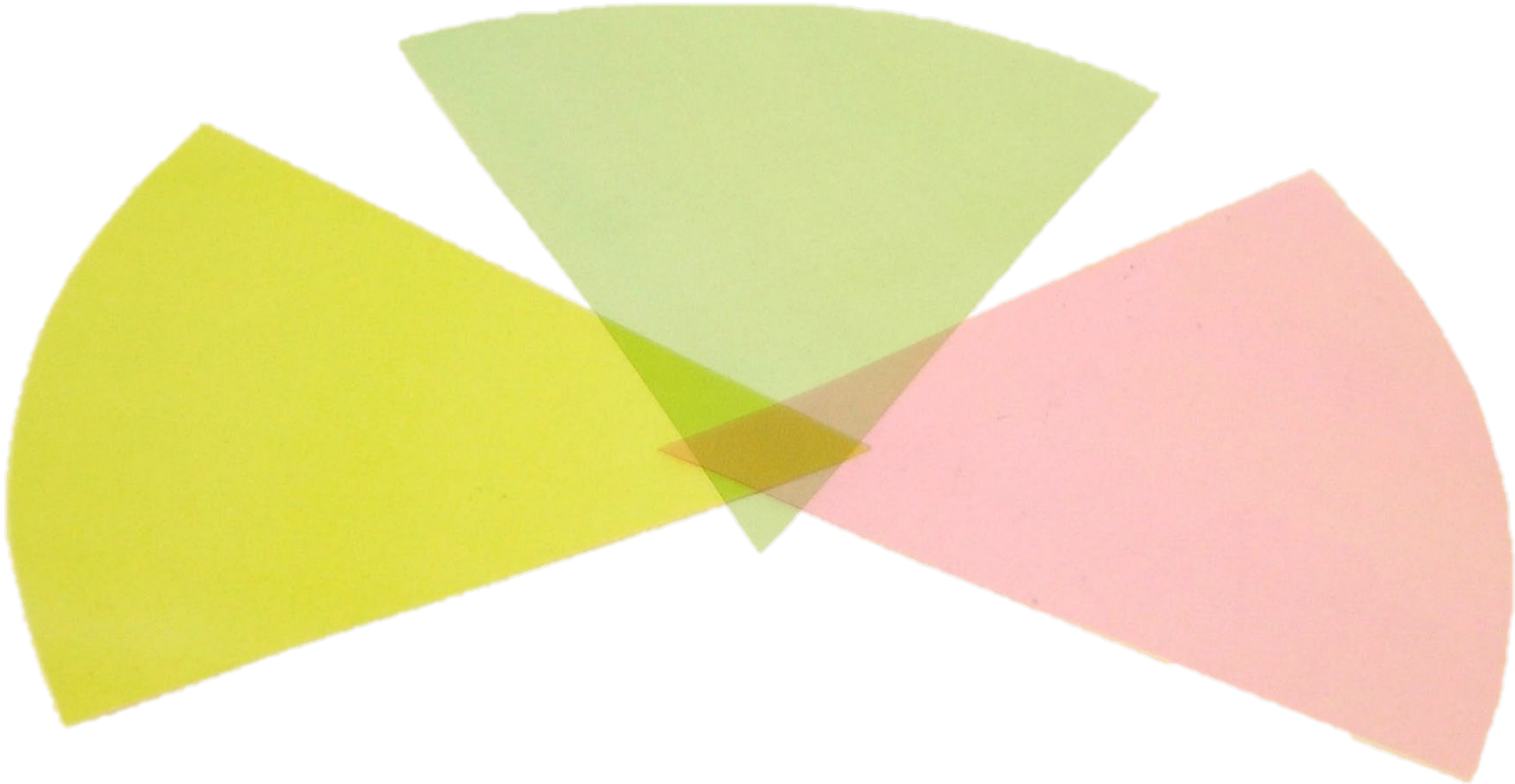


https://www.geogebra.org/m/tb5gaavx

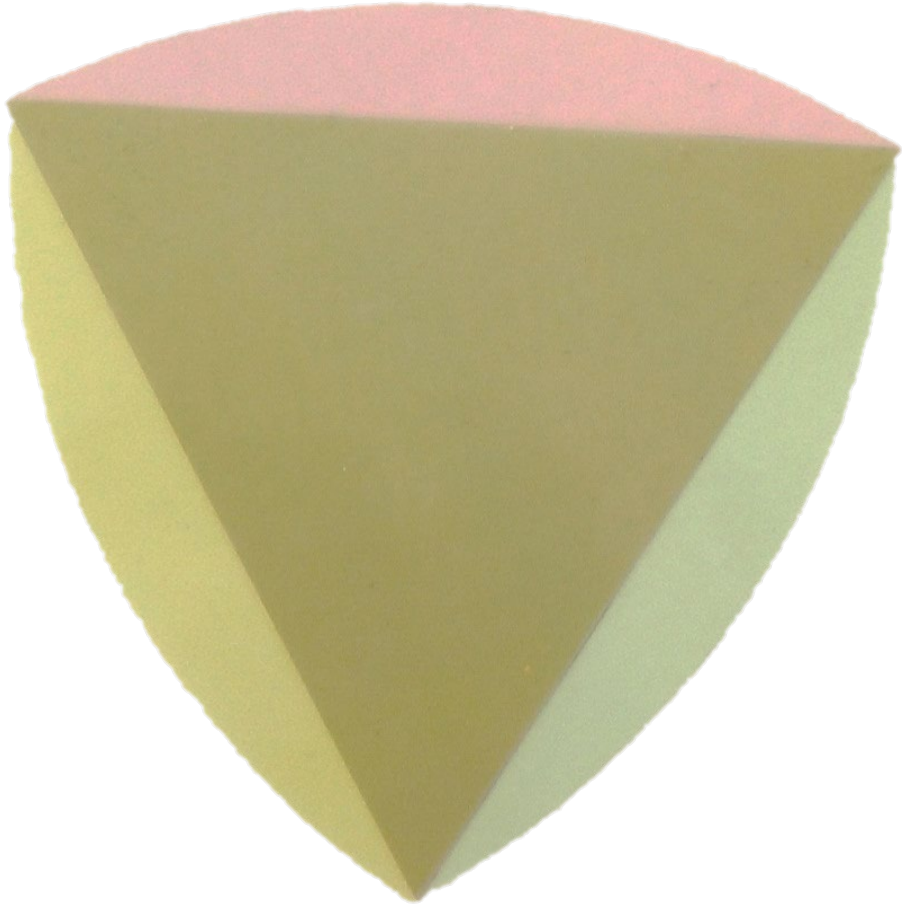
Gleichdicks gleicher Dicke: Umfang



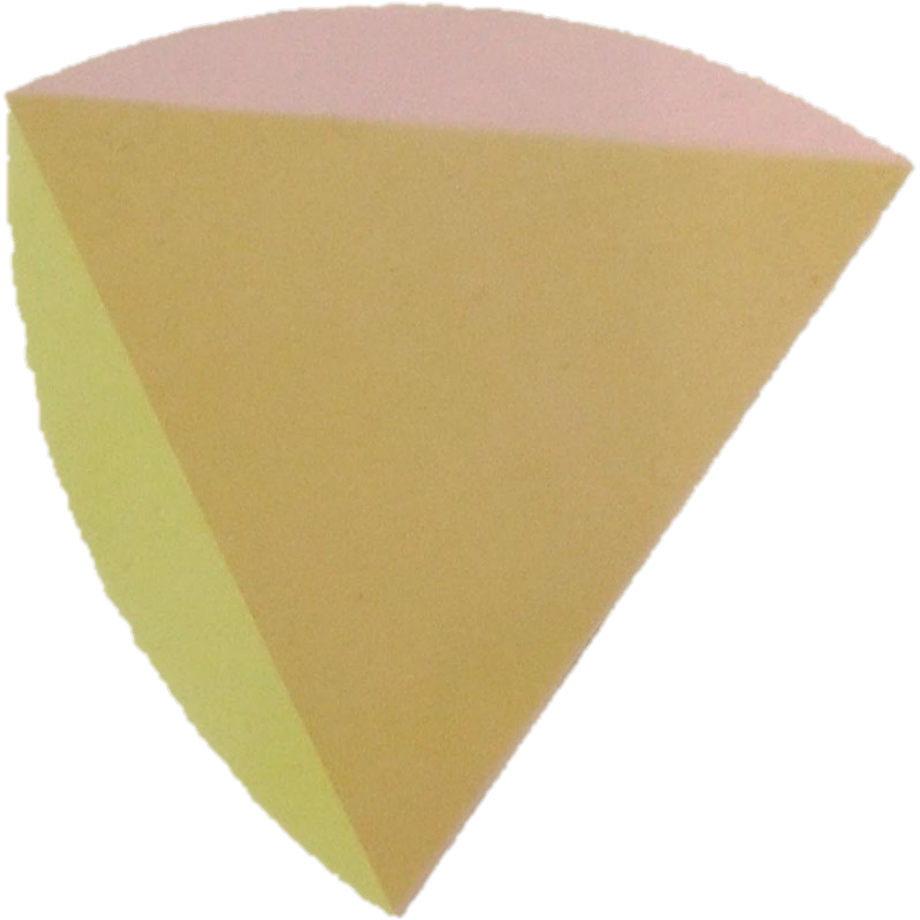
Reuleaux-Dreieck: Umfang

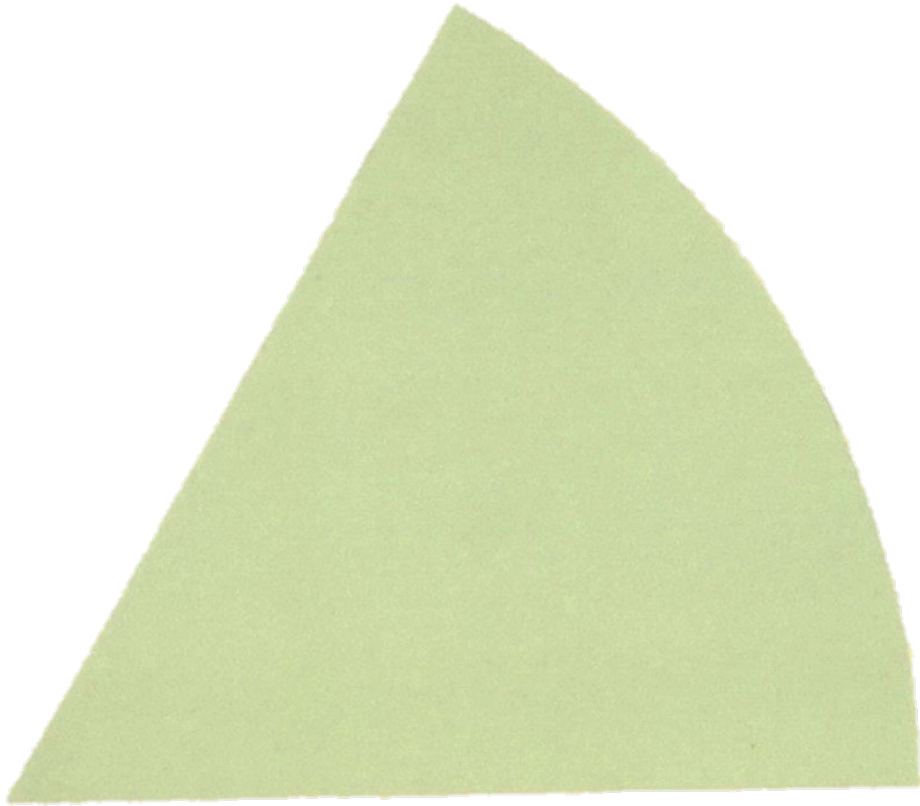


Reuleaux-Dreieck: Umfang

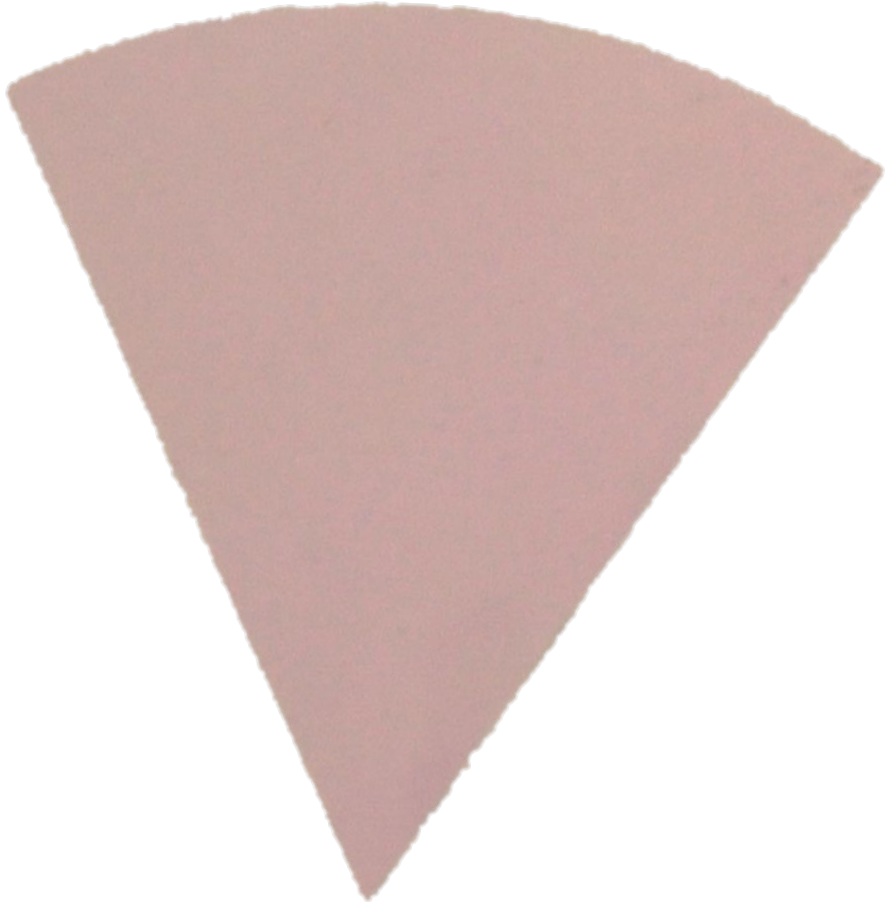


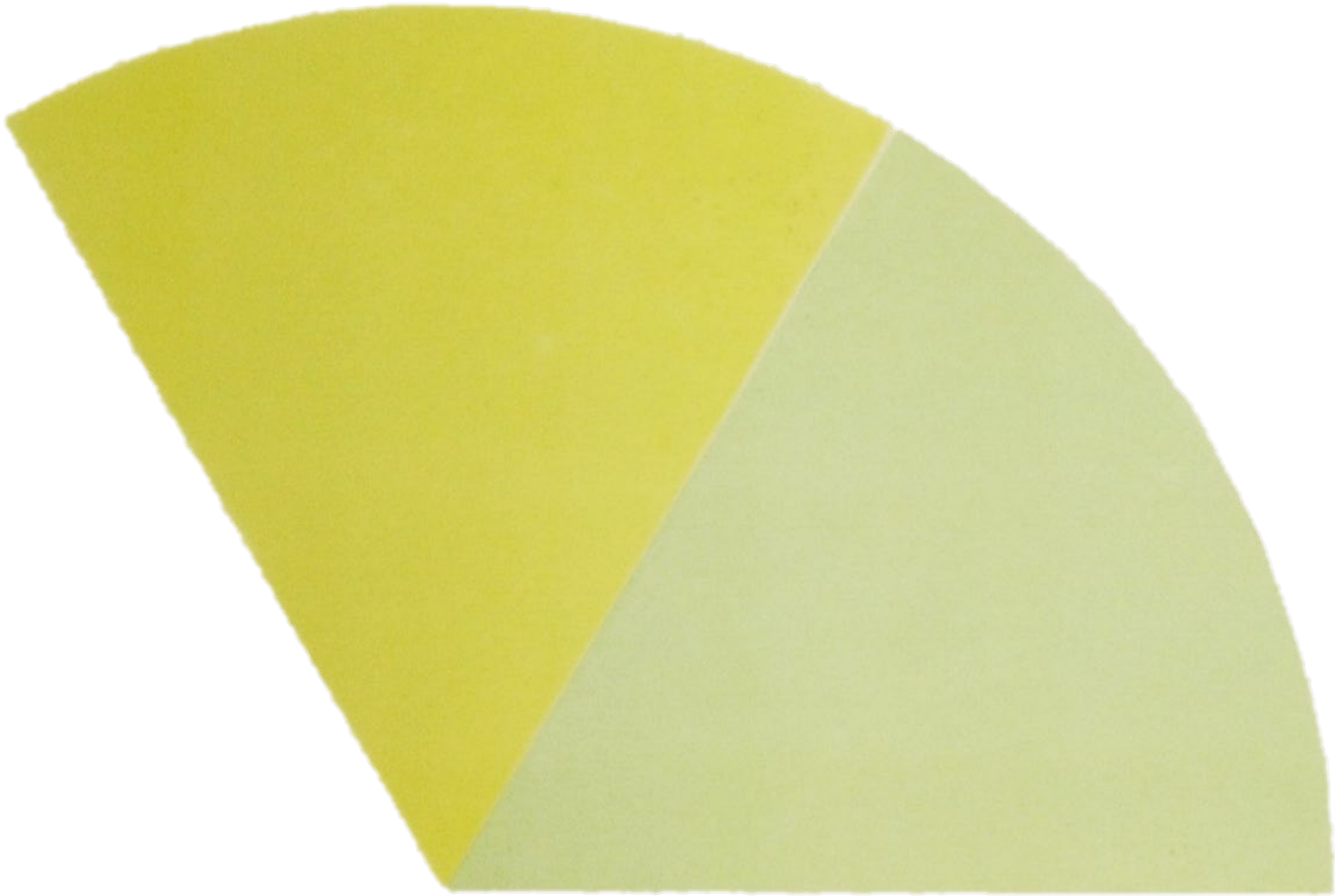
Reuleaux-Dreieck: Umfang





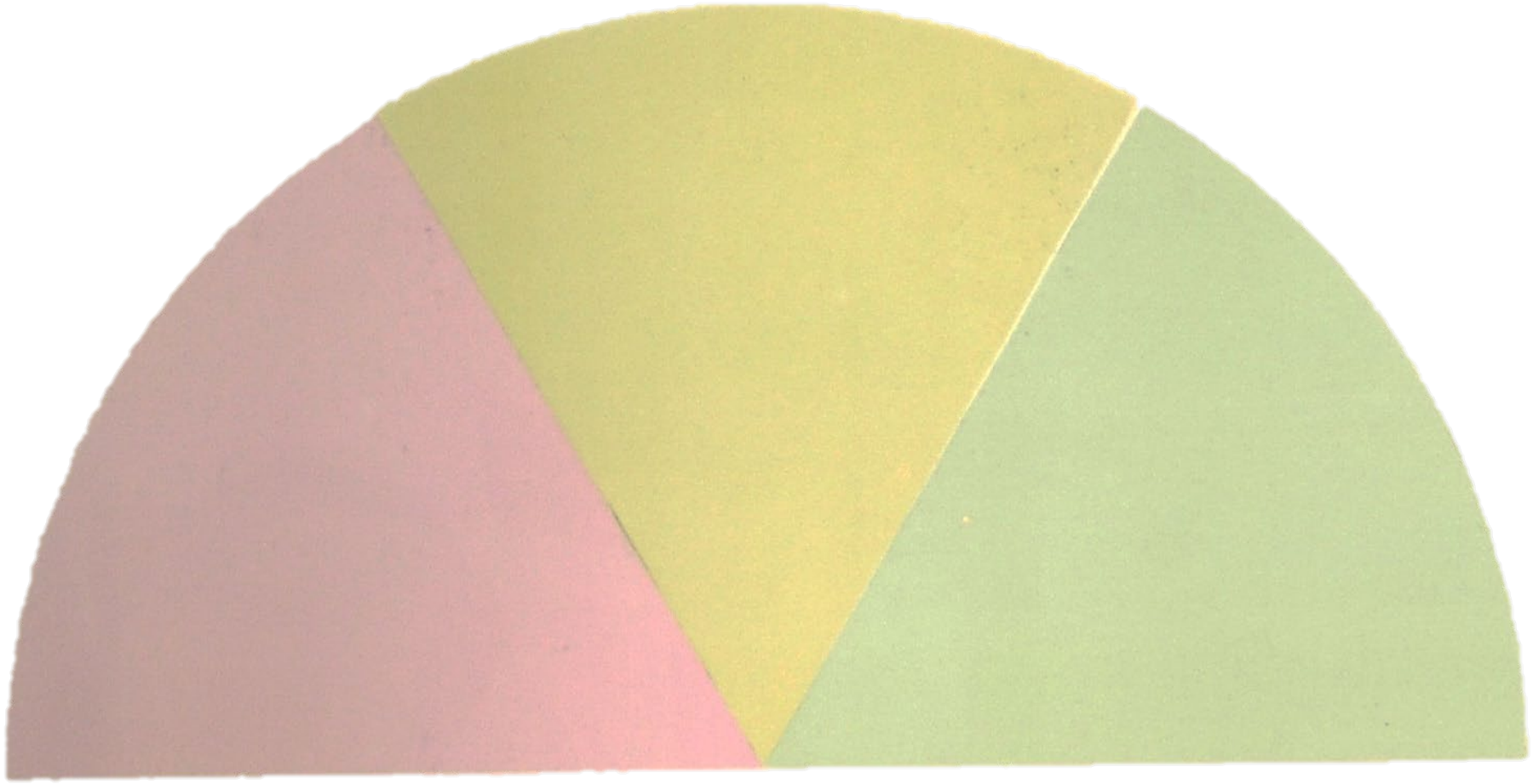
Reuleaux-Dreieck: Umfang





 https://www.geogebra.org/m/tb5gaavx

Reuleaux-Dreieck: Umfang



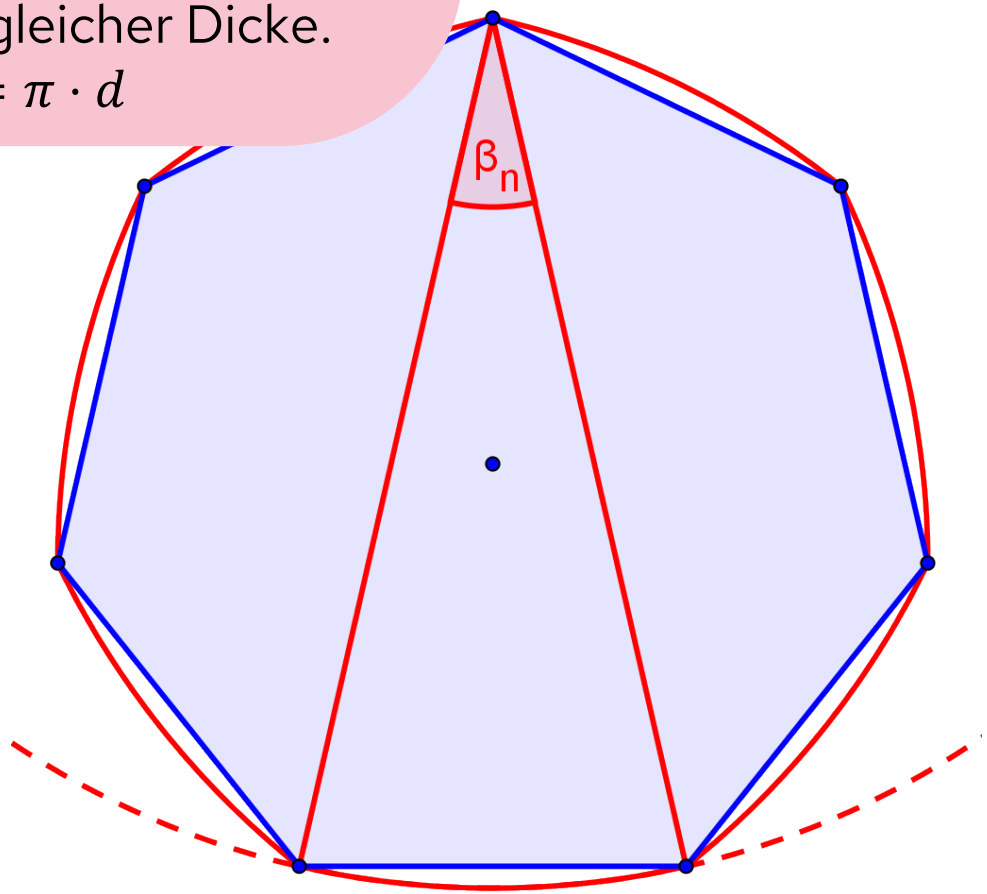
 https://www.geogebra.org/m/tb5gaavx

Reuleaux-n-Eck: Umfang

Satz von Barbier

Jedes Gleichdick der Dicke d hat den gleichen Umfang U wie der Kreis gleicher Dicke.

$$U = \pi \cdot d$$



Beweis

- Umfangswinkelsatz

$$\beta_n = \frac{1}{2} \cdot \mu_n$$

- Mit $\mu_n = \frac{1}{n} \cdot 360°$ ergibt sich:

$$\beta_n = \frac{1}{2n} \cdot 360°$$

- Kreisumfang

$$U_{Kreis} = 2\pi r = 2\pi d$$

- Länge b_n eines Kreisbogens zum Mittelpunktswinkel β_n

$$\frac{b_n}{2\pi d} = \frac{\beta_n}{360°} = \frac{1}{2n}$$

$$\Rightarrow b_n = \frac{1}{n} \cdot \pi d$$

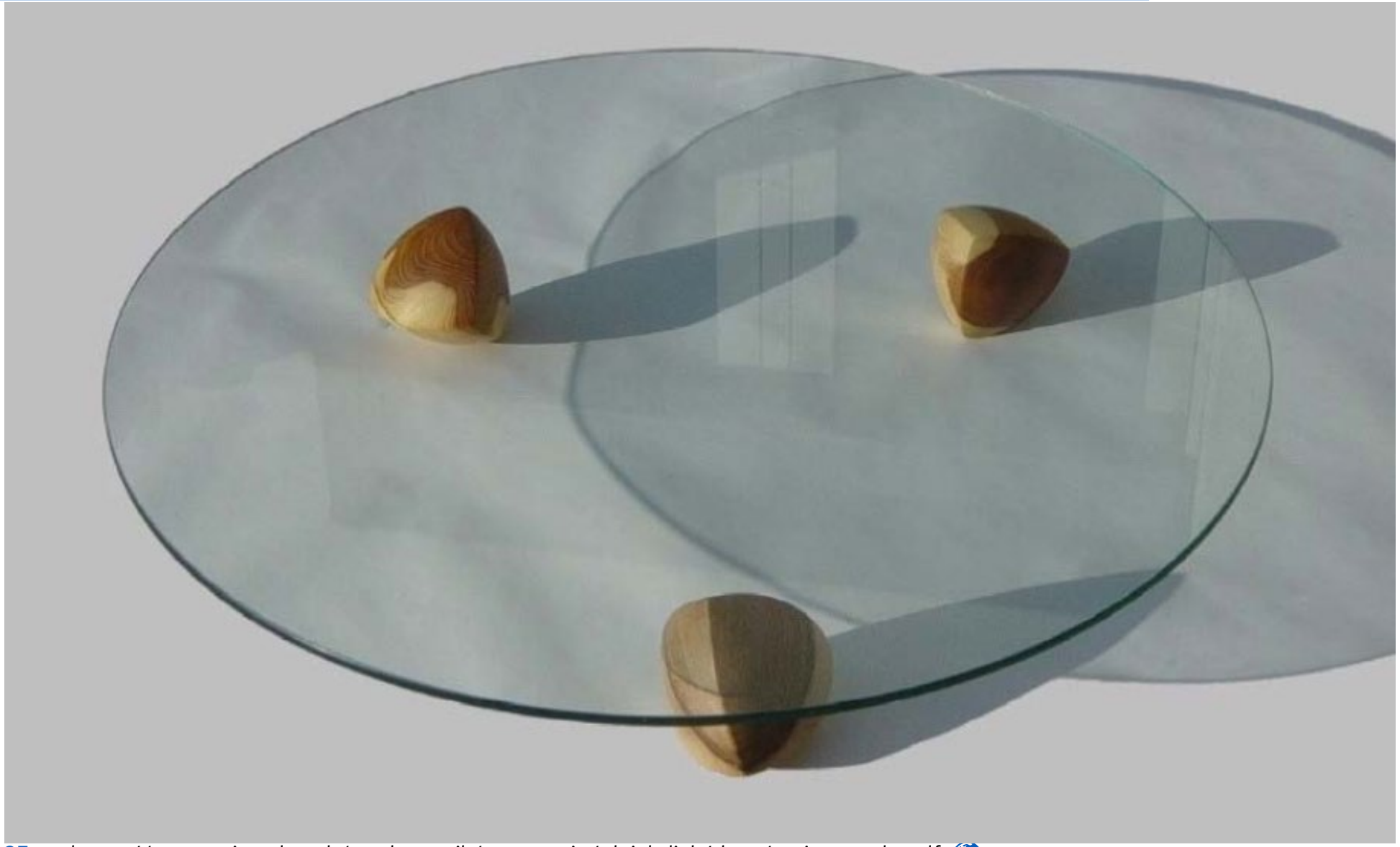
- Umfang U_n des Reuleaux-n-Ecks:

$$\boldsymbol{U_n = n \cdot b_n = \pi d}$$

2 Räumliche Gleichdicks

Reuleaux-Tetraeder

Reuleaux-Tetraeder

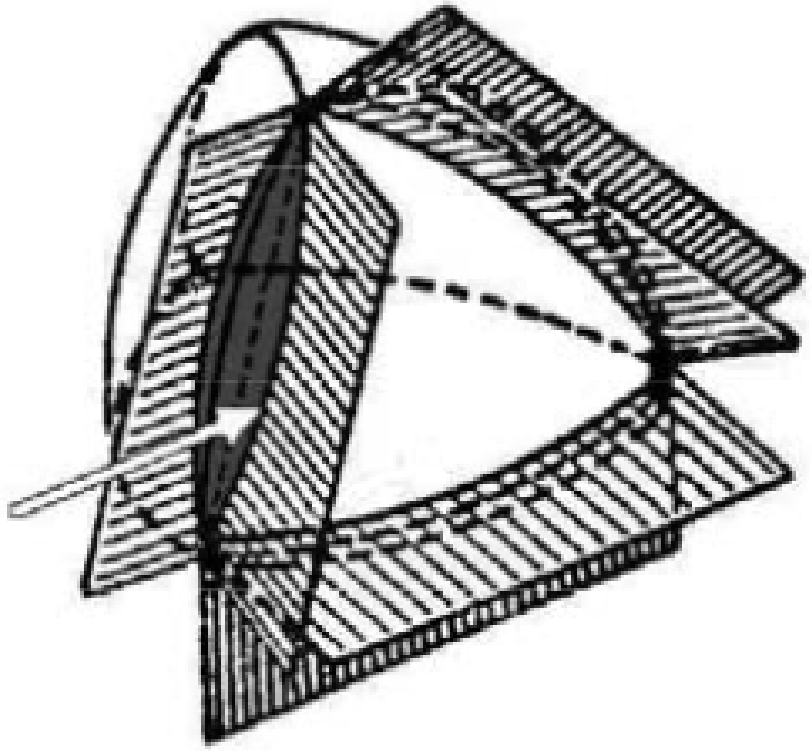


 https://www.swisseduc.ch/mathematik/geometrie/gleichdick/docs/meissner_de.pdf

Reuleaux-Tetraeder

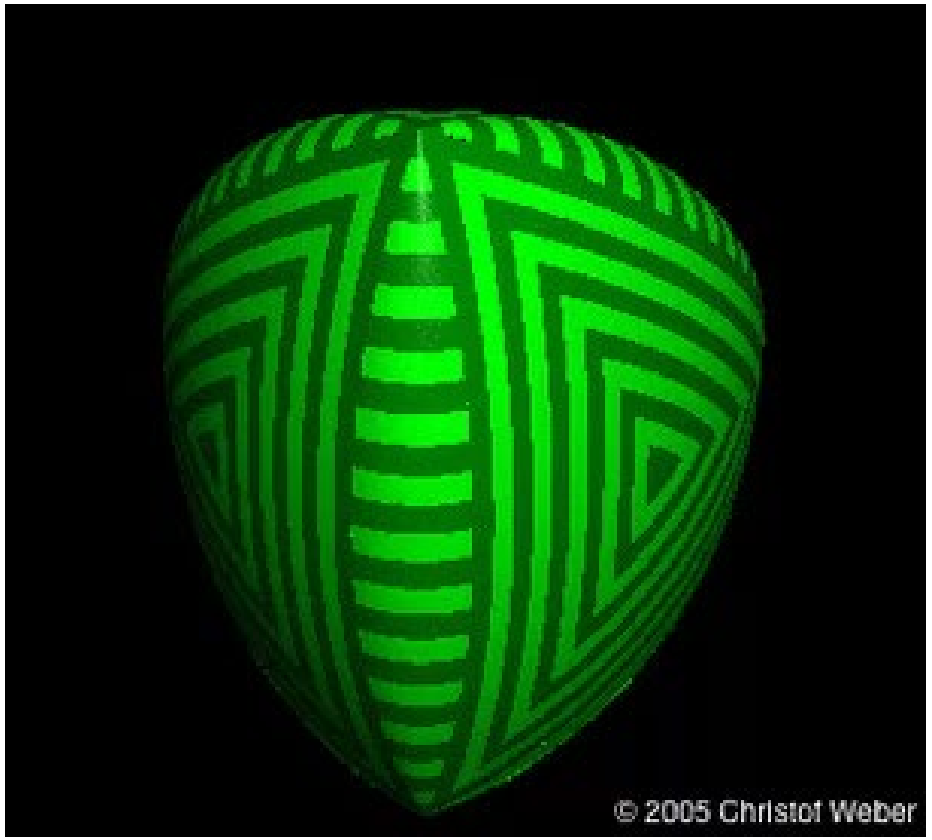
Reuleaux-Tetraeder sind nur näherungsweise Gleichdicks!

$$\sqrt{3} - \frac{\sqrt{2}}{2} \approx 1{,}0249$$

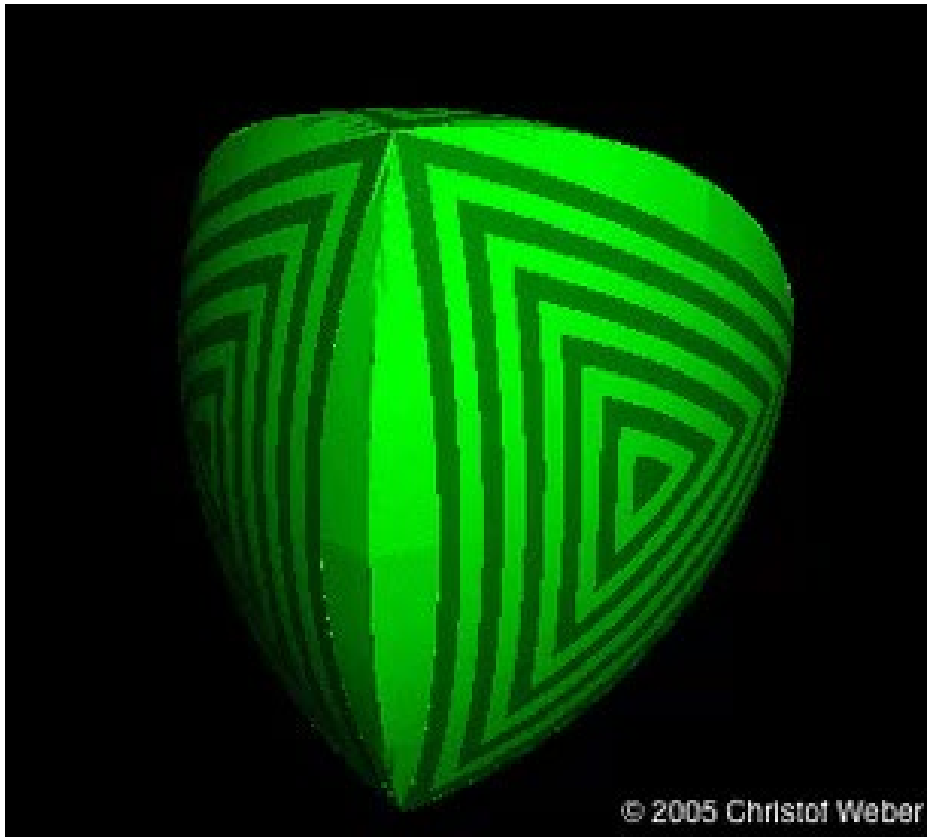


Kanten abrunden

Räumliche Gleichdicks: Meissner-Körper

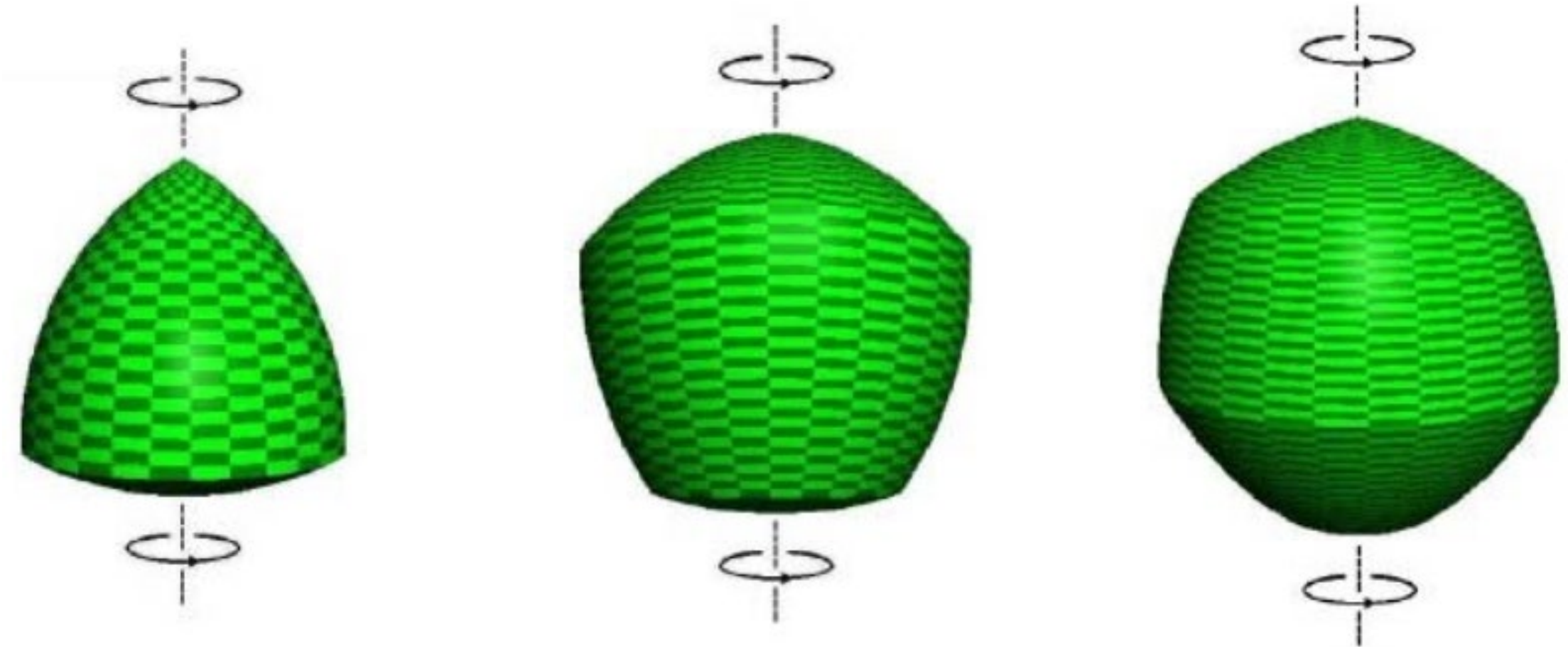


Erster Meissner-Körper
Drei Kanten, die in einer der Ecken zusammenlaufen, sind abgerundet.



Zweiter Meissner-Körper
Drei Kanten, die eine der Seiten-flächen umgeben, sind abgerundet.

Reuleaux-Tetraeder



BRYANT and
SANGWIN
How Round Is Your Circle?

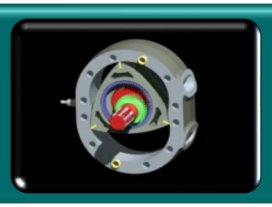
3 Anwendungen

Anwendungen

„Quadratische“ Löcher bohren

Kanaldeckel

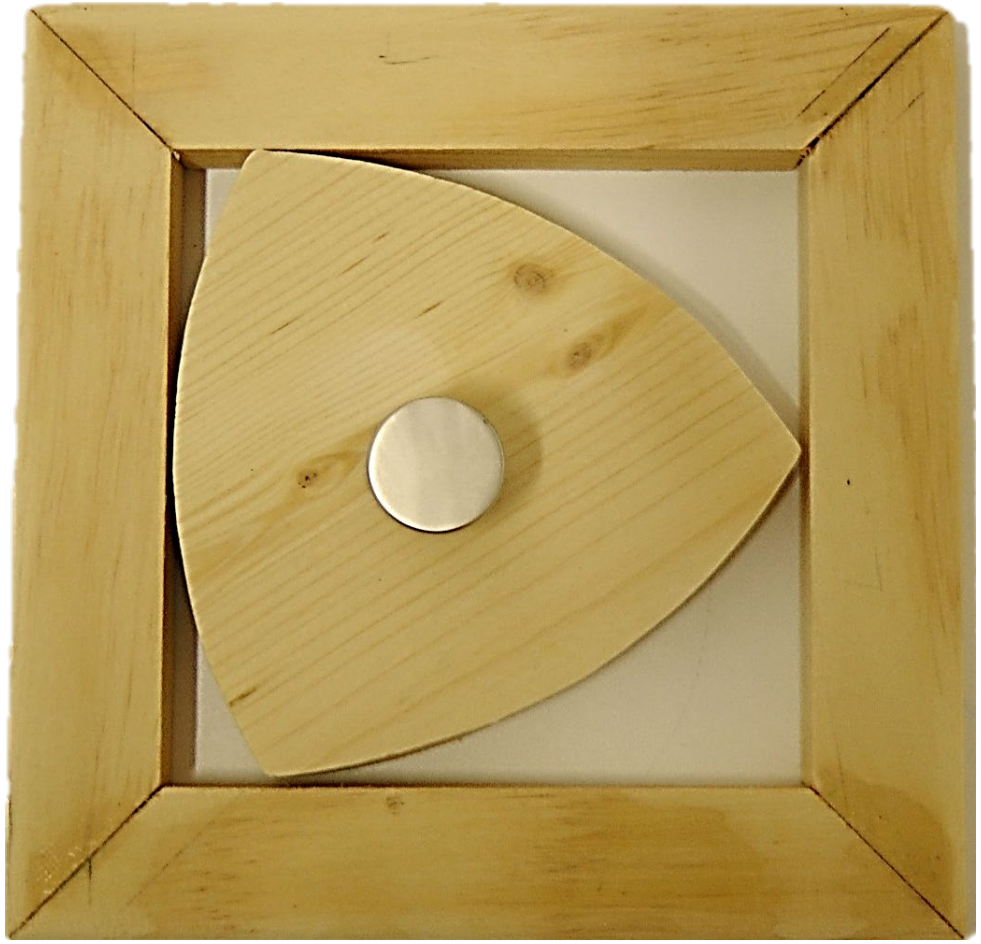
Münzen und Knöpfe

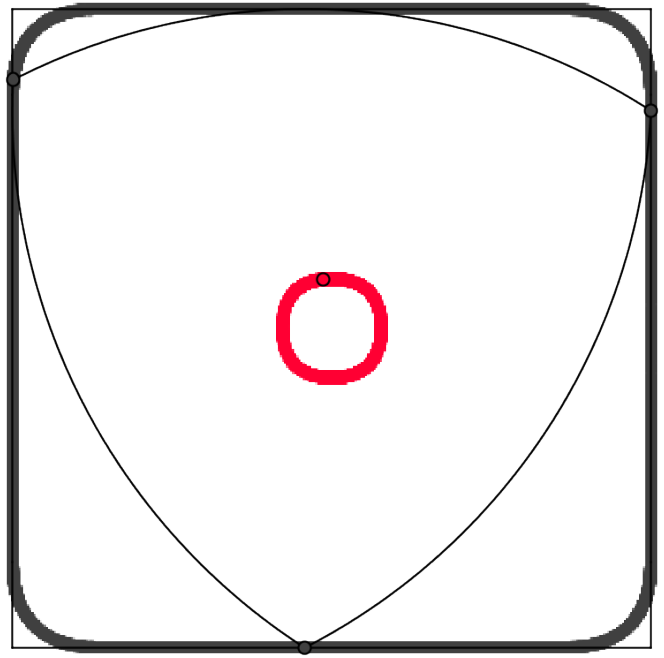


Wankelmotor

Sicherheitsverschlüsse

„Quadratische“ Löcher bohren





„Quadratische“ Löcher bohren

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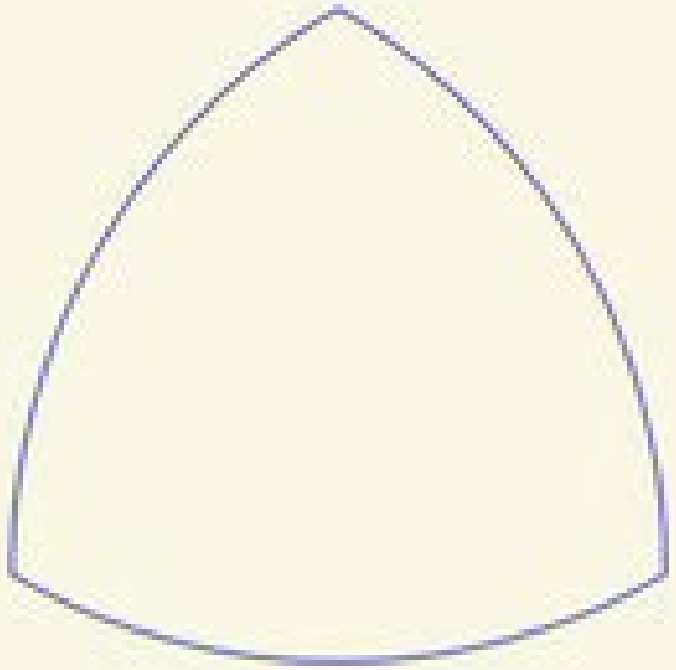

„Quadratische“ Löcher bohren

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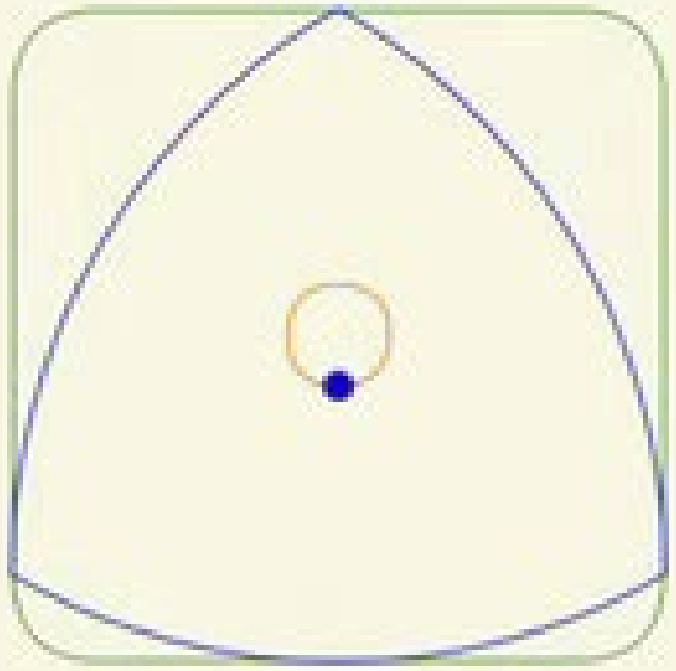

„Quadratische“ Löcher bohren

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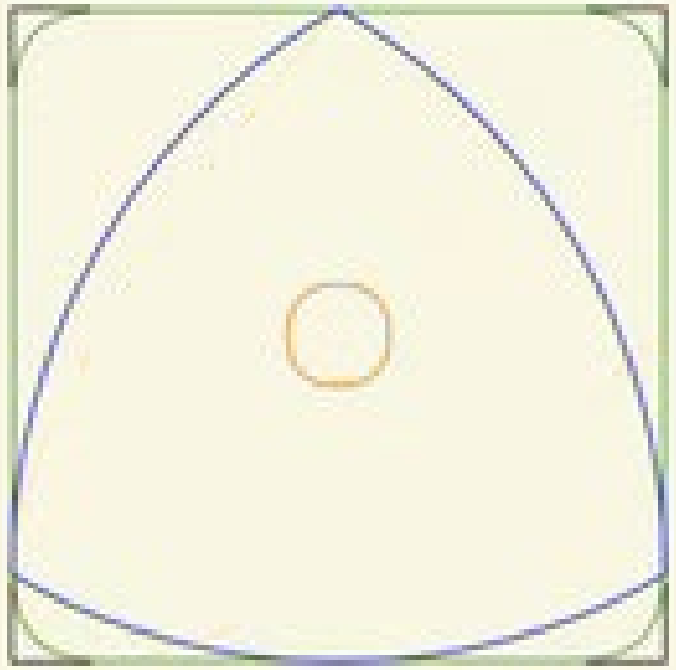
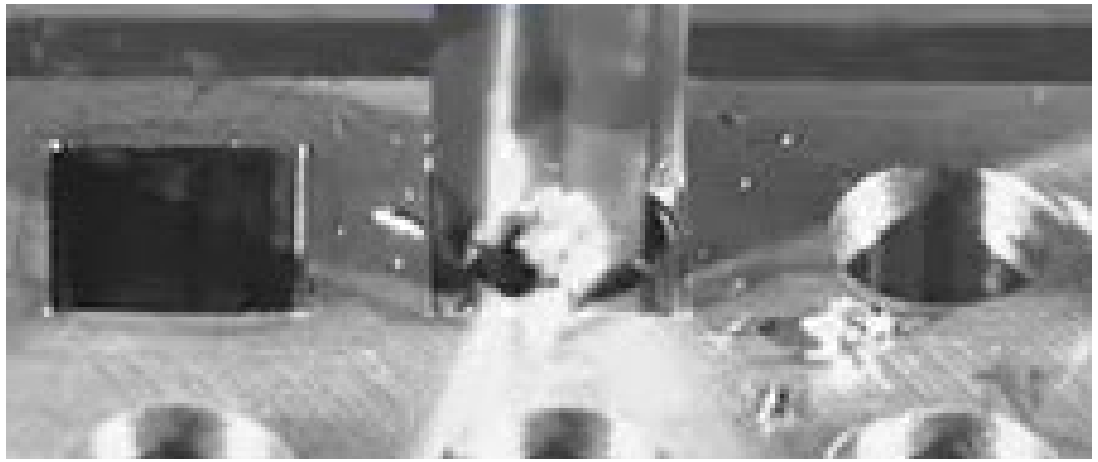


„Quadratische“ Löcher bohren



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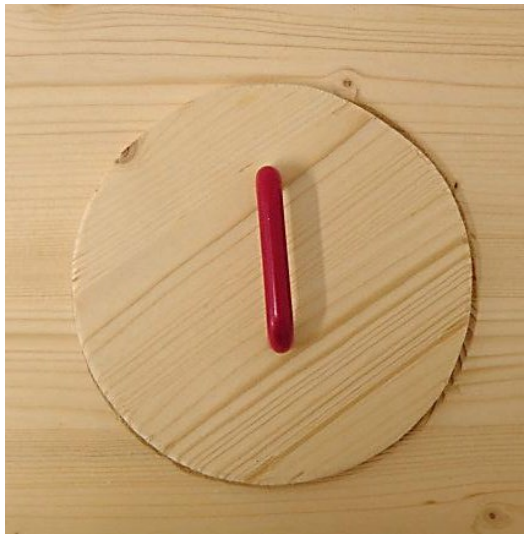
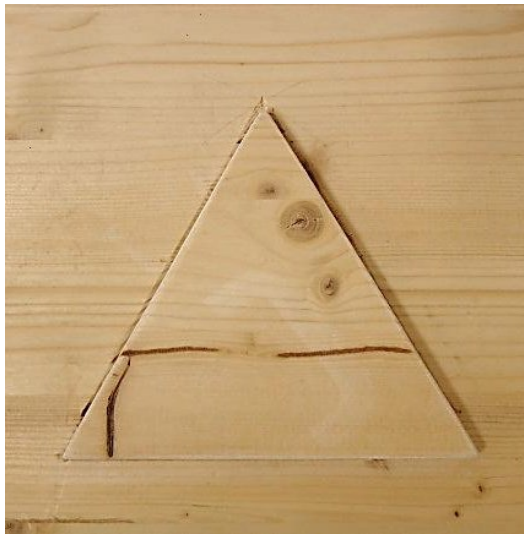
Kanaldeckel







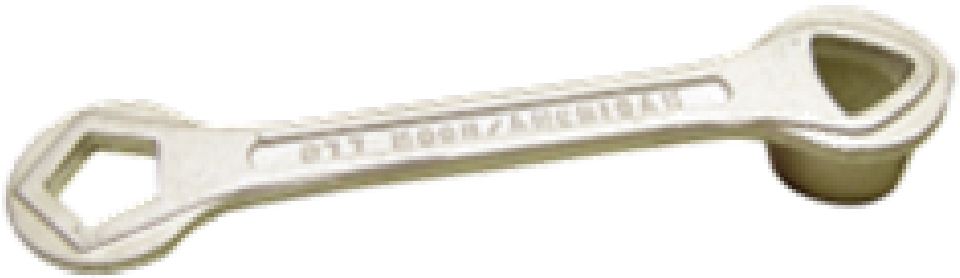

Münzen und Knöpfe

Münzen und Knöpfe

Wankelmotor

https://youtu.be/OuId4nuxXaM?si=qkOqYw5s15eJaftA

Hydranten-Verschlüsse Philadelphia (USA)



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